Completed
Geetha P.C
4/5/17

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

(Approved by AICTE & Govt of T.S and Affiliated to JNTUH)
3-5-1026, Narayanaguda, Hyderabad-29. Ph: 040-23261407

Kmit

Department Of E C E Audit Form
for
.... Course File

Faculty Name:

not in proper order arrange it properly

SNo	Topic	Audit 1	Audit 2	Audit 3
1	V / M / PEO / POs /PSOs	wrong one kept,		27/6/17
2	Course Structure	X Missing		
3	Course syllabus	not in proper place		
4	Course Outcomes (CO)	check it	?	Take new print out
5	Mapping	✓		~~II, and name~~ in pen ~~mapping to be removed~~ put it
6	Academic Calendar	X wrong		
7	Time table(class &I individual)	X missing		Take new print out ~~II not H/S~~
8	Lesson plan	of which class?		
9	Topics beyond syllabus (TBS)	✓		~~II note H/S~~ Incomplete notes
10	Web references	check it		
11	Lecture notes	✓		
12	Power point presentations / Videos	✓		
13	University Question papers	missing		2^{nd} mid?
14	Internal Question papers with Key	co, level?	key?	Students ~~Answer sheet?~~
15	Assignment Question papers	2nd Assignment	co, level?	
16	Tutorial evidence	check it		
17	Result Analysis to identify Weak and advanced learners	missing		
18	Result Analysis at the end of			

~~II Mid~~ ~~Topic Beyond Syllabus~~
Q. Soln ~~P Evidence~~

	the course			
19	Course Assessment	check it		mapping is not done
20	Guest talks, field visits etc.			[illegible]
21	Attendance register	?		Keys it
22	Course file (Digital form)			

COURSE FILE CHECK LIST FOR THE ACADEMIC YEAR 2016-17

S.NO	Topic	Remarks
1	Lesson plan	Topic wise, with references, teaching aid/methodology; Also,
2	University Question papers 15-16	3 years papers taken from exam branch
3	Internal Question papers with Key	3 years papers taken from exam branch; Answers should be written by faculty Course outcome number and Taxonomy
4	Assignment Question papers	Course outcome number and Taxonomy Level should be marked for each question
5	Result Analysis to identify Weak and advanced learners	List of Weak and advanced learners based on 1). BEFORE THE SEMESTER START: A).Students performance up to previous semester; B). Their Performance of pre-requisite course 2). AFTER 3
6	Result Analysis at the end of the course	University examination result of the previous year and the present year
7	Course Assessment	1). Mid exams marks list with attainment level calculation 2). University exam marks with attainment level calculation 3). Feedback on faculty from students –
8	Attendance register	Attendance for all students (as per Time Table) Periodic monitoring of HoD /

IQAC Committee In charge

All faculty should maintain one box file with following documents (Punched and filed) for each course taken by them during the academic year 2015-16:

Topic	Remarks
V / M / PEO / POs /PSOs	Signed copy - Xerox
Course syllabus	Preferably the University provided document
Course Outcomes (CO)	4-6 outcomes covering entire syllabus, easily explainable by the faculty (with unique numbering for each CO)
Lesson plan	Topic wise, with references, teaching aid/methodology; Also, reflect tutorials, topic beyond syllabus planning
Topics beyond syllabus (TBS)	List of topics taught other than university specified syllabus
Web references	Topic wise web links (entire topic web link) for entire syllabus
Lecture notes	Module wise, hand written and easily traceable – topic wise
Power point presentations / Videos	Presentations list (topic and file name) CD should be present in the box file itself.
University Question papers	3 years papers taken from exam branch
Internal Question papers with with Key	3 years papers taken from exam branch; Answers should be written by faculty Course outcome number and Taxonomy Level should be marked for each question
Assignment Question papers	Course outcome number and Taxonomy Level should be marked for each question
Tutorial evidence	List of tutorial topics as per time table
Result Analysis to identify Weak and advanced learners	List of Weak and advanced learners based on 1). BEFORE THE SEMESTER START: A).Students performance up to previous semester; B). Their Performance of pre-requisite course 2). AFTER 3 weeks of instruction observation 3). Based on Internal Examination marks.
Result Analysis at the end of the course	University examination result of the previous year and the present year
Course Assessment	1). Mid exams marks list with attainment level calculation 2). University exam marks with attainment level calculation 3). Feedback on faculty from students – Analysis page 4). Course outcome feedback 5). PO attainment page
Guest talks, field visits etc.	Details, if any
Attendance register	Attendance for all students (as per Time Table) Periodic monitoring of HoD / Principal Teacher log update (As per Lesson Plan) Internal marks updated
Course file (Digital form)	Page mentioning the availability of the entire course file availability to students (web site link or common location detail) All/Self-Learning materials list with the location details

VISION

MISSION

PEO/POS

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Department Of Information Technology

Vision & Mission of Department

Vision of the Department:

Producing quality graduates trained in the latest software technologies and related tools and striving to make India a world leader in software products and services.

Mission of the Department:

- Mission of the Department: To create a faculty pool which has a deep understanding and passion for algorithmic thought process.
- To impart skills beyond university prescribed to transform students into a well-rounded IT professional.
- To inculcate an ability in students to pursue Information technology education throughout their lifetime by use of multimodal learning platform including e-learning, blended learning, remote testing and skilling.
- Exposure to different domains, paradigms and exposure to the financial and commercial underpinning of the modern business environment through the entrepreneur development cell.
- To encourage collaboration with various organizations of repute for research, consultancy and industrial interactions.
- To create socially conscious and emotionally mature individuals with awareness on India's challenges, opportunities, their role and responsibility as engineers towards achieving the goal of job and wealth creation.

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Department Of Information Technology

PROGRAM OUTCOMES (POs)

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assesssocietal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

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Department Of Information Technology

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1: An ability to analyze the common business functions to design and develop appropriate Information Technology solutions for social upliftments.

PSO2: Shall have expertise on the evolving technologies like Mobile Apps, CRM, ERP, Big Data, etc.

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Department Of Information Technology

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

PEO1: Graduates will have successful careers in computer related engineering fields or will be able to successfully pursue advanced higher education degrees.

PEO2: Graduates will try and provide solutions to challenging problems in their profession by applying computer engineering principles.

PEO3: Graduates will engage in life-long learning and professional development by rapidly adapting changing work environment.

PEO4: Graduates will communicate effectively, work collaboratively and exhibit high levels of professionalism and ethical responsibility.

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Vision of the Institution:

To be the fountain head of latest technologies,
producing highly skilled, globally competent engineers.

Mission of the Institution:

- To provide a learning environment that helps students to enhance problem solving skills, be successful in their professional lives and to prepare students to be lifelong learners through multi model platforms and educating them about their professional, and ethical responsibilities.
- To establish Industry Institute Interaction to make students ready for the industry.
- To provide exposure to students to the latest tools and technologies in the area of hardware and software.
- To promote research based projects/activities in the emerging areas of technology convergence.
- To encourage and enable students to not merely seek jobs from the industry but also to create new enterprises
- To induce in the students a spirit of nationalism which will enable the student to develop and understand India's problems and to encourage them to come up with effective solutions for the same

To support the faculty in their endeavors to accelerate their learning curve in order to continue to deliver excellent service to students

COURSE STRUCTURE

With effect from 02/08/2016

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech COURSE STRUCTURE (2016-17)
(Common for EEE, ECE, CSE, EIE, BME, IT, ETE, ECM, ICE)

I YEAR I SEMESTER

S. No	Course Code	Course Title	L	T	P	Credits
1	MA101BS	Mathematics-I	3	1	0	3
2	CH102BS	Engineering Chemistry	4	0	0	4
3	PH103BS	Engineering Physics-I	3	0	0	3
4	EN104HS	Professional Communication in English	3	0	0	3
5	ME105ES	Engineering Mechanics	3	0	0	3
6	EE106ES	Basic Electrical and Electronics Engineering	4	0	0	4
7	EN107HS	English Language Communication Skills Lab	0	0	3	2
8	ME108ES	Engineering Workshop	0	0	3	2
9	*EA109MC	NSS	0	0	0	0
		Total Credits	**20**	**1**	**6**	**24**

I YEAR II SEMESTER

S. No	Course Code	Course Title	L	T	P	Credits
1	PH201BS	Engineering Physics-II	3	0	0	3
2	MA202BS	Mathematics-II	4	1	0	4
3	MA203BS	Mathematics-III	4	1	0	4
4	CS204ES	Computer Programming in C	3	0	0	3
5	ME205ES	Engineering Graphics	2	0	4	4
6	CH206BS	Engineering Chemistry Lab	0	0	3	2
7	PH207BS	Engineering Physics Lab	0	0	3	2
8	CS208ES	Computer Programming in C Lab	0	0	3	2
9	*EA209MC	NCC/NSO	0	0	0	0
		Total Credits	**16**	**2**	**13**	**24**

*Mandatory Course.

COURSE SYLLABUS

MA102BS/MA202BS: MATHEMATICS - II
(Advanced Calculus)

B.Tech. I Year II Sem.

L	T/P/D	C
4	1/0/0	4

Prerequisites: Foundation course (No prerequisites).

Course Objectives: To learn
- concepts & properties of Laplace Transforms
- solving differential equations using Laplace transform techniques
- evaluation of integrals using Beta and Gamma Functions
- evaluation of multiple integrals and applying them to compute the volume and areas of regions
- the physical quantities involved in engineering field related to the vector valued functions.
- the basic properties of vector valued functions and their applications to line, surface and volume integrals.

Course Outcomes: After learning the contents of this course the student must be able to
- use Laplace transform techniques for solving DE's
- evaluate integrals using Beta and Gamma functions
- evaluate the multiple integrals and can apply these concepts to find areas, volumes, moment of inertia etc of regions on a plane or in space
- evaluate the line, surface and volume integrals and converting them from one to another

UNIT – I

Laplace Transforms: Laplace transforms of standard functions, Shifting theorems, derivatives and integrals, properties- Unit step function, Dirac's delta function, Periodic function, Inverse Laplace transforms, Convolution theorem (without proof).
Applications: Solving ordinary differential equations (initial value problems) using Laplace transforms.

UNIT - II

Beta and Gamma Functions: Beta and Gamma functions, properties, relation between Beta and Gamma functions, evaluation of integrals using Beta and Gamma functions.
Applications: Evaluation of integrals.

UNIT – III

Multiple Integrals: Double and triple integrals, Change of variables, Change of order of integration. **Applications:** Finding areas, volumes & Center of gravity (evaluation using Beta and Gamma functions).

UNIT – IV

Vector Differentiation: Scalar and vector point functions, Gradient, Divergence, Curl and their physical and geometrical interpretation, Laplacian operator, Vector identities.

UNIT – V

Vector Integration: Line Integral, Work done, Potential function, area, surface and volume integrals, Vector integral theorems: Greens, Stokes and Gauss divergence theorems (without proof) and related problems.

Text Books:

1. Advanced Engineering Mathematics by R K Jain & S R K Iyengar, Narosa Publishers
2. Engineering Mathematics by Srimanthapal and Subodh C. Bhunia, Oxford Publishers

References:

1. Advanced Engineering Mathematics by Peter V. O. Neil, Cengage Learning Publishers.
2. Advanced Engineering Mathematics by Lawrence Turyn, CRC Press

COURSE OUTCOMES

MAPPING

ACADEMIC CALENDAR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

ACADEMIC CALENDAR (2016-17)

FOR NON-AUTONOMOUS CONSTITUENT & AFFILIATED COLLEGES

B.TECH. & B.PHARM. I & II SEMESTERS

I SEM

S. No	EVENT	DATE
1	Commencement of First Spell of Instruction	02nd Aug, 2016
2	End of First Spell of Instruction	04th Oct. 2016
3	Dussehra Vacation	05th to 12th Oct. 2016
4	First Mid Term Examinations	13th to 15th Oct. 2016
5	Commencement of Second Spell of Instruction	17th Oct. 2016
6	Submission of First Mid Term Exam Marks to University on or before	22nd Oct. 2016
7	End of Second Spell of Instruction	06th Dec. 2016
8	Second Mid Term Examinations	07th to 09th Dec 2016
9	Preparation Holidays and Practical Examinations	13th to 17th Dec. 2016
10	Submission of Second Mid Term Exam Marks to University on or before	17th Dec. 2016
11	End Semester Examinations	19th Dec. 2016 to 02nd Jan. 2017

II SEM

S. No	EVENT	DATE
1	Commencement of First Spell of Instruction	03rd Jan. 2017
2	End of First Spell of Instruction	04th Mar. 2017
3	First Mid Term Examinations	06th to 08th Mar. 2017
4	Commencement of Second Spell of Instruction	09th Mar. 2017
5	Submission of First Mid Term Exam Marks to University on or before	15th Mar. 2017
6	Parents Teacher's Meeting	18th Mar. 2017
7	End of Second Spell of Instruction	09th May 2017
8	Second Mid Term Examinations	10th to 12th May 2017
9	Preparation Holidays and Practical Examinations	15th to 20th May 2017
10	Submission of Second Mid Term Exam Marks to University on or Before	20th May 2017
11	End Semester Examinations	22nd May to 05th June 2017
12	Summer Vacation	06th Jun to 01st Jul 2017
13	Commencement of Next Academic Year (2017-18)	3rd Jul 2017

Bbhavani

DIRECTOR
ACADEMIC & PLANNING, JNTUH

* Supplementary Examinations: 9th to 16th March 2017 (Class work will be conducted during the Supplementary Examinations).

TIME TABLE

Kmit

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

Name of the faculty: Dr.T.V.A.P.SASTRY **YEAR 2016-17**

Subject: M2 , M3 **CLASS: I B.TECH- II SEM**

Period / Day	9.30-10.20	10.20-11.10	11.10-12.20	12.00 - 12.50	12.50-1.45	1.45-2.35	2.35-3.25	3.25-4.15
	1	2	3	4		5	6	7
Mon	CSE C	CSE G			LUNCH	IT	CSE G (TUT)	CSE G
Tue	CSE G	CSE C		IT				IT
Wed	IT	IT (MENTORING)				CSE G	IT (TUT)	CSE C
Thu	CSEC (TUT)		IT	CSE C			CSE G	
Fri				IT		CSE G	CSE C	
Sat	CSE C							

M2 - IT
CSE - C, CSE - G :M3

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

Narayanaguda, Hyderabad-29

DEPARTMENT OF INFORMATION TECHNOLOGY

B.Tech I -Year II Sem. Time Table (2016-17) **W.E.F:03.01.2017**

TIME/DAY	9.30-10.20 1	10.20-11.10 2	11.10-12.00 3	12.00-12.50 4	12.50-1.45	1.45-2.35 5	2.35-3.25 6	3.25-4.15 7
MON	ENGINEERING GRAPHICS		CP	M-3	L	M-2	EG	M-3
TUE	M-3	EP-2	*EP/M-2	M-2	U	M-3	EP-2	M-2
WED	M-2	MENTORING& COUNSEELING	CP	EP-2	N	CP LAB		
THU	*EG/M-3	M-3	M-2	EP-2	C	EP LAB		
FRI	M 3	CP	EP-2	M-2	H	ENGINEERING GRAPHICS		
SAT	CP	EC LAB				CP	*M-2/M3	EP-2

*- Tutorial

SL.NO	NAME OF THE SUBJECT	NAME OF THE FACULTY
1	MATHEMATICS-2 (M-2)	DR. SHASTRY
2	MATHEMATICS-3 (M-3)	MD.YOUNUS
3	ENGINEERING PHYSICS-2 (EP-2)	MR. D.SRINIU
4	COMPUTER PROGRAMMING (CP)	MR.NEIL GOGTE (IT)
5	ENGINEERING GRAPHICS (EG)	MR. PATIL
6	ENGINEERING CHEMISTRY LAB	MS. HEMANGI JOSHI/MS N JAGRUTHI
7	ENGINEERING PHYSICS LAB	MR.SRINU & MS. M.UDAYA LAKSHMI
8	COMPUTER PROGRAMMING LAB	MR.NEIL GOGTE/ MS.POOJA DIXIT (IT)
9	MENTORING & COUNSELLING	DR. T.V.A.P. SASTRY/MS. HEMANGI JOSHI/MS. SHARMEELA CHUNGI

CLASS-INCHARGE **HOD** **PRINCIPAL**

PRINCIPAL
Keshav Memorial Institute of Technology
Narayanaguda, Hyderabad - 500 029.

LESSON PLAN

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY
DEPARTMENT OF INFORMATION TECHNOLOGY

Subject Code	Name of the Subject	Class/ Sem	Name of the Faculty / Designation	No. of Students	Total Proposed Periods per semester/year	
					Lectures	Tutorial
MA202BS	Mathematics-II	I YEAR ECE-B	Dr.T.V.A.P.SASTRY Asso. Prof.	60	92	7

WEEK Number	Lecture Number	Topic	Date of Completion	Textbook/ References	Teaching aid/methodology
Wk1	1	**UNIT-I** **Laplace transforms** Introduction Laplace transforms of standard functions, linearity property	3/1	HIGHER ENGG MATHS BY B.S. Grewal	White Board
	2	Problems of Laplace transform	4/1	"	White Board
	3	First shifting theorem and its problems, unit step function	5/1	"	White Board
	4	Second shifting theorem , change of scale property and their problems	6/1	"	White Board
	7	Laplace transforms of derivatives , integrals and their problems	7/1	"	White Board
	8	TUTORIAL	7/1	"	White Board
	9	Laplace transforms multiplication with t and related problems	9/1	"	White Board
Wk 2	10	Laplace transforms of division with t and related problems	10/1	"	White Board
	11	Dirac delata function , and its laplace transform, periodic function and its laplace transform and related problems	11/1	"	White Board
	12	Problems on evaluation of integrals by laplace transform	12/1	"	White Board
	13	Revision of periodic functions and evaltuation of integrals.	16/1	"	White Board
	14	Inverse Laplace transforms and basic formulae, few problems	17/1	"	White Board
	15	Problems on inverse laplace transforms .	18/1	"	White Board
	16	First shifting, second shifting, change of scale and related problems	19/1	"	White Board

Signature of the HOD
Date

Signature of the Faculty
Date

*This column has to be filled-up after completion of the lecture/tutorial/practical in the copy kept with the faculty members.

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY
DEPARTMENT OF INFORMATION TECHNOLOGY

	17	Revision of all the above topic of inverse laplace transform.	20/1	"	White Board
Wk 3	18	Inverse laplace transform of derivatives and integrals and related problems	21/1	"	White Board
	18	Inverse laplace transform of mult and division with s and related problems	21/1	"	White Board
	19	Problems on all the above 4 topics	23/1	"	White Board
Wk 4	20	Convolution theorem and related problems	24/1	"	White Board
	21	Problems related to Convolution theorem	25/1	"	White Board
	22	Solving ODE by Laplace transforms	27/1	ENGG MATHS BY SRIMANTPAL OXFORD PUBLSHERS	White Board
	23	Solving ODE by Laplace transforms	28/1	"	White Board
	24	TUTORIAL	28/1	"	White Board
		UNIT-II		"	White Board
	25	Introduction to improper integrals and beta functions	30/1	"	White Board
Wk5	26	Properties of Beta function and evaluation	31/1	"	White Board
	27	Forms of beta function	1/2	"	White Board
	28	Forms of beta function	2/2	"	White Board
	29	Solving problems on beta functions	3/2	"	White Board
	30	Solving problems on beta functions	4/2	"	White Board
	31	Definition of gamma function, few formulae, Properties of Gamma function	6/2	"	White Board
	32	beta gamma relation , few problems	7/2	"	White Board

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	33	Evaluation of integrals using Beta and Gamma functions	8/2	"	White Board
	34	Evaluation of integrals using Beta and Gamma functions	9/2	"	White Board
	35	Evaluation of integrals using Beta and Gamma functions	10/2	"	White Board
	36	Evaluation of integrals using Beta and Gamma functions	13/2	"	White Board
	37	TUTORIAL	14/2	"	White Board
	38	Revision	15/2	"	White Board
Wk7	39	Evaluation of integrals using Beta and Gamma functions	16/2	"	White Board
		UNIT-III		"	White Board
	40	Introduction to Multiple integrals, Double integrals in Cartesian coordinates	17/2	"	White Board
	41	Evaluation of double integrals in Cartesian coordinates	18/2	"	White Board
	42	Evaluation of double integrals in Cartesian coordinates	20/2	"	White Board
Wk8	43	Evaluation of double integrals in polar coordinates	21/2	"	White Board
	44	Evaluation of double integrals in polar coordinates	22/2	"	White Board
	45	Conversion of Cartesian to polar and vice versa and related problems	23/2	"	White Board
	46	Change of order of integration method and related problems	25/2	"	White Board
	47	TUTORIAL	25/2	"	White Board
	48	Change of order of Integration problems	27/2	"	White Board
wk 9	49	Change of order of Integration problems	28/2	"	White Board
	50	Revision	1/3	"	White Board
	51	Triple integrals in Cartesian formulae	2/3	"	White Board
	52	Problems on triple integrals	3/3	"	White Board

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	53	Revision	4/3	"	White Board
wk 11	54	Triple integrals in spherical and cylindrical formulae and problems	14/3	P&S FOR ENGINEERS BY DEVORE	White Board
	55	Evaluation of triple integrals in spherical and cylindrical	15/3	"	White Board
	56	Conversion of triple integrals	16/3	HIGHER ENGG MATHS BY B.S. Grewal	White Board
	57	Finding areas using double integrals	17/3	"	White Board
wk 12	58	Finding volumes using double integrals	20/3	"	White Board
	59	Volume of a region using triple Integration.	21/3	"	White Board
	60	Finding the center of gravity using Beta and Gamma functions	22/3	"	White Board
	61	Revision	23/3	"	White Board
		UNIT-IV		"	White Board
	62	**Vector differentiation**: introduction to vectors , few defns like gradient, curl, and divergence	24/3	"	White Board
	63	Gradient of a Scalar point function, few formulae and related problems	25/3	"	White Board
	64	Problems on above topic	25/3	"	White Board
wk 13	65	Directional derivative problems		"	White Board
	66	Angle between normals	27/3	"	White Board
	67	Divergence of vector point function and few problems	28/3	"	White Board
	68	Solinoidal problems and problems related to divergence	30/3	"	White Board
	69	Curl of a vector function and related problems	31/3	"	White Board
	70	Irrotational of a vector function	1/4	"	White Board

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		and other problems			
wk 14	71	Laplacian operator and related problems	3/4	"	White Board
	72	Vector identities	4/4	"	White Board
	73	Problems on Vector identities	6/4	"	White Board
	74	TUTORIAL	7/4	"	White Board
		UNIT-V		"	White Board
wk 15	75	Vector Integration Introduction, line integrals and related formulae	10/4	"	White Board
	76	Line integral – work done problems	11/4	"	White Board
	77	Problems on line integrals	12/4	"	White Board
	78	Problems on line integrals	13/4	"	White Board
	79	Surface integrals and related formulae and problems	15/4	"	White Board
	80	Problems on Surface integrals	17/4	"	White Board
wk 16	81	Problems on Surface integrals	18/4	"	White Board
	82	Volume integrals and related formulae	19/4	"	White Board
	83	Problems on Volume integrals	20/4	"	White Board
	84	Problems on Volume integrals	21/4	"	White Board
	85	Vector integral theorems introduction and Greens theorem statement	22/4	"	White Board
	86	TUTORIAL	22/4	"	White Board
	87	Problems on Greens theorem	24/4	"	White Board
	88	Problems on greens theorem	25/4	"	White Board
	89	Revision	26/4	"	White Board
	90	Stroke theorem and related problems	27/4	"	White Board

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wk 17	91	Problems on Stroke theorem	28/4	"	White Board
	92	Problems on strokes theorem	29/4	"	White Board
	93	Gauss divergence theorem statement and related problems	1/5	"	White Board
wk 18	94	Problems on Gauss divergence theorem	2/5	"	White Board
	95	Problems on guass divergence theorem	3/5	"	White Board
	96	revision	4/5	"	White Board
	97	revision	6/5		White Board
	98	revision	8/5		
	99	revision	9/5		

Signature of the HOD
Date

Signature of the Faculty
Date

*This column has to be filled-up after completion of the lecture/tutorial/practical in the copy kept with the faculty members.

TOPIC BEYOND SYLLABUS

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

DEPARTMENT OF INFORMATION TECHNOLOGY

TOPICS BEYOND THE SYLLABUS

SUB: Mathematics-II

YEAR/SEM: I/II **CLASS: IT**

S.NO	Unit	Topics beyond the syllabus	Text book/web reference
1	I	Solving PDE using Laplace Transforms	Web reference
2	II	Applications of Beta functions	R1
3	III	Applications of Multiple Integrals	T2
4	IV	Application of Vectors	T1
5	V	Gauss divergence theorem proof, Green theorem proof and Stokes theorem proof	R1

A Proof of Stoke's Theorem

The proof of Stoke's theorem follows directly from Green's theorem. Before doing so, however, let's noti
and if we let $\mathbf{F}_1 = \langle M,0,0 \rangle$, $\mathbf{F}_2 = \langle 0,N,0 \rangle$, and $\mathbf{F}_3 = \langle 0,0,P \rangle$, then $\mathbf{F} = \mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3$ and similarly

$$curl(\mathbf{F}) = curl(\mathbf{F}_1) + curl(\mathbf{F}_2) + curl(\mathbf{F}_3)$$

As a result, Stoke's theorem is proved if we can show it for each of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$. Below we prove Stok
for $\mathbf{F}_2$ and $\mathbf{F}_3$ are left to the exercises.

Suppose that $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$ which maps a region S in the uv-plane to a surface Σ in R^3
S is mapped to the boundary of Σ. If $\partial\Sigma$ is parameterized by $\mathbf{r}(t)$ for t in $[a,b]$, then the work integral bec

$$\oint_{\partial\Sigma} \mathbf{F}_1 \cdot d\mathbf{r} = \int_a^b \mathbf{F}_1 \cdot \frac{d\mathbf{r}}{dt}\, dt$$

$$= \int_a^b \mathbf{F}_1 \cdot \left(\frac{\partial \mathbf{r}}{\partial u}\frac{du}{dt} + \frac{\partial \mathbf{r}}{\partial v}\frac{dv}{dt} \right) dt$$

$$= \int_a^b \mathbf{F}_1 \cdot \mathbf{r}_u \frac{du}{dt} + \mathbf{F} \cdot \mathbf{r}_v \frac{dv}{dt}\, dt$$

$$= \oint_{\partial S} \mathbf{F}_1 \cdot \mathbf{r}_u\, du + \mathbf{F}_1 \cdot \mathbf{r}_v\, dv$$

That is, we can pull the boundary curve back into the uv-plane. However, Green's theorem in the uv-plane ir

$$\oint_{\partial S} \mathbf{F}_1 \cdot \mathbf{r}_u\, du + \mathbf{F}_1 \cdot \mathbf{r}_v\, dv = \iint_S \frac{\partial}{\partial u}(\mathbf{F}_1 \cdot \mathbf{r}_v) - \frac{\partial}{\partial v}(\mathbf{F}_1 \cdot \mathbf{r}_u)\, dud$$

If we let $\mathbf{F}_{1,u}$ denote the partial of $\mathbf{F}_1$ with respect to u – i.e., $\mathbf{F}_{1,u} = \partial_u \mathbf{F}_1$ – then

$$\oint_{\partial\Sigma} \mathbf{F}_1 \cdot d\mathbf{r} = \iint_S \frac{\partial}{\partial u}(\mathbf{F}_1 \cdot \mathbf{r}_v) - \frac{\partial}{\partial v}(\mathbf{F}_1 \cdot \mathbf{r}_u)\, dudv$$

$$= \iint_S \mathbf{F}_{1,u} \cdot \mathbf{r}_v + \mathbf{F}_1 \cdot \mathbf{r}_{uv} - \mathbf{F}_1 \cdot \mathbf{r}_{uv} - \mathbf{F}_{1,v} \cdot \mathbf{r}_v\, dudv$$

$$= \iint_S \mathbf{F}_{1,u} \cdot \mathbf{r}_v - \mathbf{F}_{1,v} \cdot \mathbf{r}_u\, dudv$$

Substituting $\mathbf{F}_1 = \langle M,0,0 \rangle$, $\mathbf{r}_u = \langle x_u, y_u, z_u \rangle$, and $\mathbf{r}_v = \langle x_v, y_v, z_v \rangle$ results in

$$\oint_{\partial\Sigma} \mathbf{F}_1 \cdot d\mathbf{r} = \iint_S \langle M_u,0,0 \rangle \cdot \langle x_v, y_v, z_v \rangle - \langle M_v,0,0 \rangle \cdot \langle x_u, y_u, z_u \rangle\ d$$

$$= \iint_S (M_u x_v - M_v x_u)\, dudv$$

Our goal now is to show that the integrand of the double integral is $curl(\mathbf{F}_1) \cdot d\mathbf{S}$. To do so, we first use the chai
$M_x x_u + M_y y_u + M_z z_u$ and $M_v = M_x x_v + M_y y_v + M_z z_v$ to obtain

$$\oint_{\partial\Sigma} \mathbf{F}_1 \cdot d\mathbf{r} = \iint_S [(M_x x_u + M_y y_u + M_z z_u)\, x_v - (M_x x_v + M_y y_v + M_z z_v)\, x_u]\ d$$

$$= \iint_S [(M_y y_u + M_z z_u)\, x_v - (M_y y_v + M_z z_v)\, x_u]\, dudv$$

$$= \iint_S [M_y y_u x_v + M_z z_u x_v - M_y y_v x_u - M_z z_v x_u]\, dudv$$

Rearranging terms and factoring out M_z and M_y yields

$$\oint_{\partial\Sigma} \mathbf{F}_1 \cdot d\mathbf{r} = \iint_S [M_z(z_u x_v - z_v x_u) - M_y(x_u y_v - x_v y_u)]\, dudv$$

$$= \iint_S M_z \begin{vmatrix} z_u & x_u \\ z_v & x_v \end{vmatrix} - M_y \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix}\, dudv$$

Since $curl(\mathbf{F}_1) = \langle 0, M_z, -M_y \rangle$, we rewrite the integrand as

$$\mathbf{F}_1 \cdot d\mathbf{r} = \iint \langle 0, M_z, -M_y \rangle \cdot \langle \begin{vmatrix} y_u & z_u \end{vmatrix}, \begin{vmatrix} z_u & x_u \end{vmatrix}, \begin{vmatrix} x_u & y_u \end{vmatrix}$$

$$\oint_{\partial\Sigma} \qquad \int\int_S \qquad \left\langle \left| \; y_v \quad z_v \right| \left| \; z_v \quad x_v \right| \left| \; x_v \quad y_v \right| \right\rangle$$

$$= \int\int_S curl(\mathbf{F}_1) \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dudv$$

$$= \int\int_\Sigma curl(\mathbf{F}_1) \cdot d\mathbf{S}$$

The proof for $\mathbf{F}_2$ and $\mathbf{F}_3$ is similar, so that

$$\int\int_\Sigma curl(\mathbf{F}) \cdot d\mathbf{S} = \oint_{\partial\Sigma} \mathbf{F}_1 \cdot d\mathbf{r} + \oint_{\partial\Sigma} \mathbf{F}_2 \cdot d\mathbf{r} + \oint_{\partial\Sigma} \mathbf{F}_3 \cdot d\mathbf{r}$$

$$= \oint_{\partial\Sigma} (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3) \cdot d\mathbf{r}$$

$$= \oint_{\partial\Sigma} \mathbf{F} \cdot d\mathbf{r}$$

thus proving Stoke's theorem.

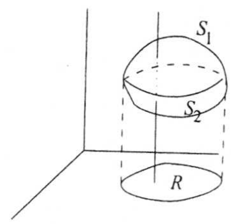
V10.2 The Divergence Theorem

2. Proof of the divergence theorem.

We give an argument assuming first that the vector field $\mathbf{F}$ has only a $\mathbf{k}$-component: $\mathbf{F} = P(x, y, z)\,\mathbf{k}$. The theorem then says

$$\text{(4)} \qquad \iint_S P\,\mathbf{k}\cdot\mathbf{n}\,dS = \iiint_D \frac{\partial P}{\partial z}\,dV\,.$$

The closed surface S projects into a region R in the xy-plane. We assume S is vertically simple, i.e., that each vertical line over the interior of R intersects S just twice. (S can have vertical sides, however — a cylinder would be an example.) S is then described by two equations:



$$\text{(5)} \qquad z = g(x, y) \quad \textit{(lower surface)}; \qquad z = h(x, y) \quad \textit{(upper surface)}$$

The strategy of the proof of (4) will be to reduce each side of (4) to a double integral over R; the two double integrals will then turn out to be the same.

We do this first for the triple integral on the right of (4). Evaluating it by iteration, we get as the first step in the iteration,

$$\text{(6)} \qquad \begin{aligned} \iiint_D \frac{\partial P}{\partial z}\,dV &= \iint_R \int_{g(x,y)}^{h(x,y)} \frac{\partial P}{\partial z}\,dz\,dx\,dy \\ &= \iint_R \big(P(x,y,h) - P(x,y,g)\big)\,dx\,dy \end{aligned}$$

To calculate the surface integral on the left of (4), we use the formula for the surface area element $d\mathbf{S}$ given in V9, (13):

$$d\mathbf{S} = \pm(-z_x\,\mathbf{i} - z_y\,\mathbf{j} + k)\,dx\,dy,$$

where we use the $+$ sign if the normal vector to S has a positive k-component, i.e., points generally upwards (as on the upper surface here), and the $-$ sign if it points generally downwards (as it does for the lower surface here).

This gives for the flux of the field $P\mathbf{k}$ across the upper surface S_2, on which $z = h(x, y)$,

$$\iint_{S_2} P\,\mathbf{k}\cdot d\mathbf{S} = \iint_R P(x, y, z)\,dx\,dy = \iint_R P(x, y, h(x, y))\,dx\,dy\,.$$

while for the flux across the lower surface S_1, where $z = g(x, y)$ and we use the $-$ sign as described above, we get

$$\iint_{S_1} P\,\mathbf{k}\cdot d\mathbf{S} = \iint_R -P(x, y, z)\,dx\,dy = \iint_R -P(x, y, g(x, y))\,dx\,dy\,;$$

adding up the two fluxes to get the total flux across S, we have

$$\iint_S P\,\mathbf{k}\cdot d\mathbf{S} = \iint_R P(x, y, h)\,dx\,dy - \iint_R P(x, y, g)\,dx\,dy$$

which is the same as the double integral in (6). This proves (4). □

In the same way, if $\mathbf{F} = M(x,y,z)\,\mathbf{i}$ and the surface is simple in the $\mathbf{i}$ direction, we can prove

$$(4') \qquad \iint_S M\mathbf{i}\cdot\mathbf{n}\,dS = \iiint_D \frac{\partial M}{\partial x}\,dV$$

while if $\mathbf{F} = N(x,y,z)\,\mathbf{j}$ and the surface is simple in the $\mathbf{j}$ direction,

$$(4'') \qquad \iint_S N\mathbf{j}\cdot\mathbf{n}\,dS = \iiint_D \frac{\partial N}{\partial y}\,dV\,.$$

Finally, for a general field $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ and a closed surface S which is simple in all three directions, we have only to add up (4), (4′), and (4″). and we get the divergence theorem.

If the domain D is not bounded by a closed surface which is simple in all three directions, it can usually be divided up into smaller domains D_i which are bounded by such surfaces S_i. Adding these up gives the divergence theorem for D and S, since the surface integrals over the new faces introduced by cutting up D each occur twice, with the opposite normal vectors $\mathbf{n}$, so that they cancel out; after addition, one ends up just with the surface integral over the original S.

Prove. Let P(x, y) and Q(x, y) be continuous and have continuous partial derivatives in a region R and on its boundary C. Then

$$1)\quad \oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$$

Proof. There are some difficulties in proving Green's theorem in the full generality of its statement. However, for regions of sufficiently simpl shape the proof is quite simple. We will prove it for a simple shape and then indicate the method used for more complicated regions. We will require such a shape that lines parallel to either x or y axis cut the boundary C of the region at no more that two points.

We shall prove the following two statements:

$$2)\quad \int_C P\,dx = -\iint_R \frac{\partial P}{\partial y}\,dx\,dy$$

$$3)\quad \int_C Q\,dy = \iint_R \frac{\partial Q}{\partial x}\,dx\,dy$$

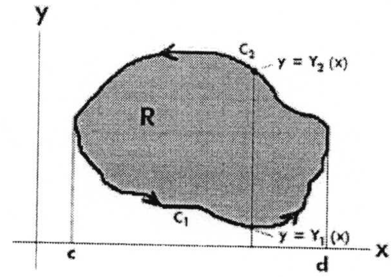


Fig. 1

This will conclude the proof since the sum of 2) and 3) gives 1).
We shall now proceed to prove 2) and shall utilize Fig. 1. Let the boundary of region R consist of a lower curve $y = Y_1(x)$ and an upper curve $y = Y_2(x)$ as shown. Let c_1 and c_2 denote the lower and upper curves. Then

$$\int_C P\,dx = \int_{C_1} P\,dx + \int_{C_2} P\,dx$$

Computing the line integral for C_1

$$\int_{C_1} P(x,y)\,dx = \int_c^d P(x, Y_1(x))\,dx$$

where c and d are the limits shown in the figure.

Similarly, for C_2 we have

$$\int_{C_2} P(x,y)\,dx = \int_d^c P(x, Y_2(x))\,dx = -\int_c^d P(x, Y_2(x))\,dx$$

Thus

$$4)\quad \int_C P\,dx = -\int_c^d \{P(x, Y_2) - P(x, Y_1)\}\,dx$$

Now let us consider the double integral in the right member of 2). It can be written as

$$5)\quad \iint_R \frac{\partial P}{\partial y}\,dx\,dy = \int_c^d \left[\int_{Y_1}^{Y_2} \frac{\partial P}{\partial y}\,dy \right] dx$$

By the Fundamental Theorem of Integral Calculus the integral within the brackets can be written as

$$6)\quad \int_{Y_1}^{Y_2} \frac{\partial P}{\partial y}\,dy = P(x, Y_2) - P(x, Y_1)$$

and 5) becomes

$$7)\quad \iint_R \frac{\partial P}{\partial y}\,dx\,dy = \int_c^d \{P(x, Y_2) - P(x, Y_1)\}\,dx$$

From 4) and 7) we get

8) $$\int_C P dx = -\iint_R \frac{\partial P}{\partial y}\, dx\, dy$$

which is 2) above.

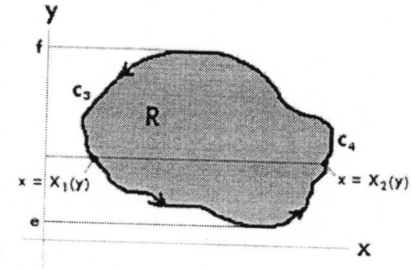


Fig. 2

In a completely similar way we can obtain 3) above using Fig. 2. Then adding 2) and 3) we get 1).

How do we extend the proof of the theorem to more complicated shapes? We divide the more complicated shapes up into simpler regions of the type we have just considered using cuts such as the cut MN shown in Fig. 3. These cuts add to the boundary traversed by the amount of the cuts, traversed twice in opposite directions. Because they are traversed in opposite directions, the line integrals along the cuts cancel each other out, the net boundary traversed remains the same, and the theorem remains unchanged. More explicitly, referring to Fig. 3 we have

9) $$\int_{MNLM} P\,dx + Q\,dy = \iint_{R_1} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx dy$$

10) $$\int_{MKNM} P\,dx + Q\,dy = \iint_{R_2} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx dy$$

Adding the left sides of 9) and 10), omitting the integrands P dx + Q dy, we get

$$\int_{MNLM} + \int_{MKNM} = \int_{MN} + \int_{NLM} + \int_{MKN} + \int_{NM} = \int_{NLM} + \int_{MKN} = \int_{NLMKN}$$

using the fact that

$$\int_{MN} = -\int_{NM}$$

Adding the left sides of 9) and 10), omitting the integrands, we get

$$\iint_{R_1} + \iint_{R_2} = \iint_{R}$$

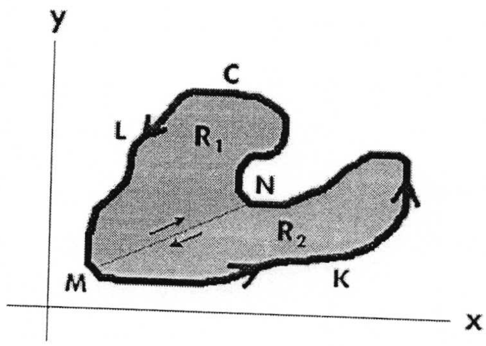


Fig. 3

Consequently

$$\int_{NLMKN} P dx + Q dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy$$

For more complicated regions we may need to construct more cuts dividing the region into more subregions.

Suppose the region is multiply-connected as shown in Fig. 4. How do we extend the proof to multiply-connected regions? For this we create a **cross-cut** MN, connecting the exterior and interior boundaries as shown in the figure, thus converting the region into a simply-connected region. The amount of boundary traversed is increased by the cross-cut MN, traversed in opposite directions. Because it is traversed in opposite directions the line integrals on the cross-cut cancel each other out and the net boundary traversed remains the same, namely c_1 plus c_2, and the theorem remains the same.

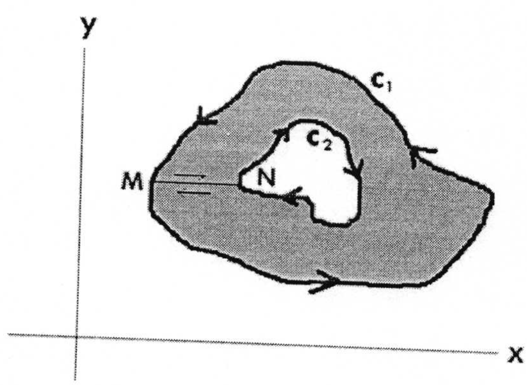


Fig. 4

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Beta Function and its Applications

Riddhi D.
*Department of Physics and Astronomy
The University of Tennessee
Knoxville, TN 37919, USA

Abstract

The Beta function was first studied by Euler and Legendre and was given its name by Jacques Binet. Just as the gamma function for integers describes factorials, the beta function can define a binomial coefficient after adjusting indices. The beta function was the first known scattering amplitude in string theory, first conjectured by Gabriele Veneziano. It also occurs in the theory of the preferential attachment process, a type of stochastic urn process. The incomplete beta function is a generalization of the beta function that replaces the definite integral of the beta function with an indefinite integral. The situation is analogous to the incomplete gamma function being a generalization of the gamma function.

1 Introduction

The beta function $\beta(p,q)$ is the name used by Legendre and Whittaker and Watson(1990) for the beta integral (also called the Eulerian integral of the first kind). It is defined by

$\beta(p,q) = \frac{(p-1)!(q-1)}{(p+q-1)!}$

To derive the integral representation of the beta function, we write the product of two factorial as

$m!n! = \int_0^\infty e^{-u}u^m du \int_0^\infty e^{-v}v^n dv.$

Now taking $u = x^2, v = y^2$,so

$m!n! = 4\int_0^\infty e^{-u^2}x^{2m+1}dx \int_0^\infty e^{-v^2}y^{2n+1}dy$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}|x|^{2m+1}|y|^{2n+1}dxdy.$

Transforming to polar coordinates with $x = r\cos\theta, y = r\sin\theta$

$m!n! = \int_0^{2\pi}\int_0^\infty e^{-v^2}|r\cos\theta|^{2m+1}|r\sin\theta|^{2n+1}rdrd\theta$

to get

$m!n! = 2(m+n+1)!\int_0^{\pi/2}\cos^{2m+1}\theta\sin^{2n+1}\theta d\theta$

The beta function is then defined by

$\beta(m+1,n+1) = 2\int_0^{\pi/2}\cos^{2m+1}\theta\sin^{2n+1}\theta d\theta$

$= \frac{m!n!}{(m+n+1)!}$

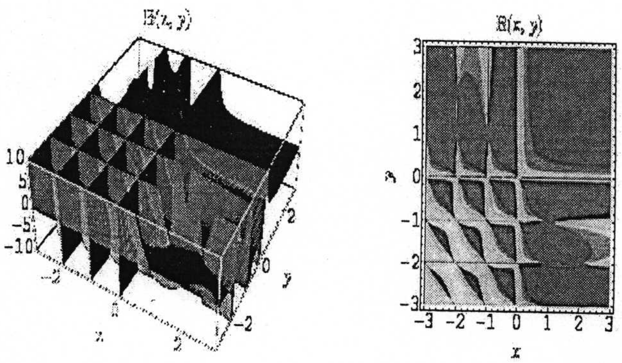
Rewriting the arguments then gives the usual form for the beta function,

$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \frac{(p-1)!(q-1)}{(p+q-1)!}$

The general trigonometric form is

$\int_0^{\pi/2}\sin^n x\cos^m x dx = \frac{1}{2}\beta\{\frac{1}{2}(n+1), \frac{1}{2}(m+1)\}$

[1][2][5]



3-D Image of Beta Function

1.1 *Beta Integral:-*

$\beta_a(x) = \int_0^1 t^{a-1}(1-t)^{x-1}dt,$

is called the Eulerian integral of the first kind by Legendre and Whittaker and Watson(1990).The solution of this integral is the Beta function $\beta(p+1,q+1)$. [1][2]

The Beta function evolves from Gamma function

$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \frac{(p-1)!(q-1)}{(p+q-1)!}$

where Γsignifies the Gamma function.

Relationship between the Gamma function and the Beta function can be derived as
$\Gamma(x)\Gamma(y) = \int_0^\infty e^{-u}u^{x-1}du \int_0^\infty e^{-v}v^{y-1}dv.$

Now taking $u = a^2, v = b^2, so$
$\Gamma(x)\Gamma(y) = 4\int_0^\infty e^{-a^2}a^{2x+1}da \int_0^\infty e^{-b^2}b^{2y+1}db$
$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(a^2+b^2)}\,|a|^{2x-1}\,|b|^{2y-1}\,dadb.$

Transforming to polar coordinates with
$a = r\cos\theta, b = r\sin\theta$
$\Gamma(x)\Gamma(y) = \int_0^{2\pi}\int_0^\infty e^{-r^2}\,|r\cos\theta|^{2x-1}\,|r\sin\theta|^{2y-1}\,rdrd\theta$ $=$
$\int_0^\infty e^{-r^2}r^{2x+2y-2}rdr \int_0^{2\pi}\left|(\cos\theta)^{2x-1}(\sin\theta)^{2y-1}\right|d\theta$
$= \Gamma(x+y)\beta(x,y)$

Hence, rewriting the arguments with the usual form of Beta function:
$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$

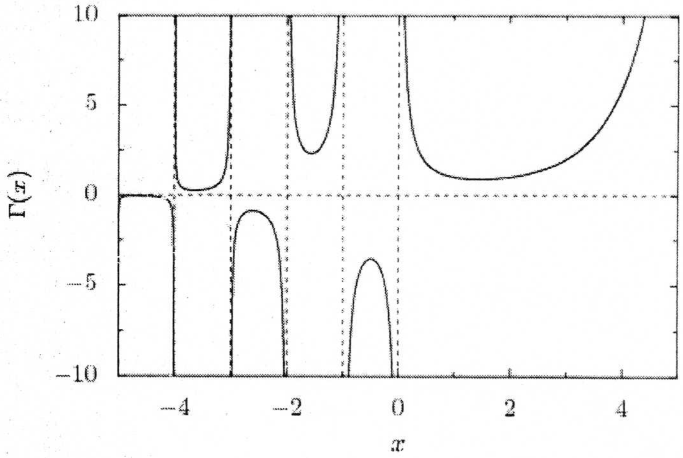
A somewhat more straightforward derivation :
$\Gamma(x)\Gamma(y) = \int_0^\infty t^{x-1}e^{-t}dt \int_0^\infty s^{y-1}e^{-s}ds$
$= \int_{t=0}^{\infty}\int_{s=0}^{\infty} t^{x-1}s^{y-1}e^{-(t+s)}dsdt$

The argument in the exponential inspires us to employ the substitution
$\sigma = s + t$
$\tau = t$
Thus$|J| = 1$,
where J is the Jacobian of the transformation. Using this transformation,
$\Gamma(x)\Gamma(y) = \int_{\sigma=0}^{\infty}\int_{\tau=0}^{\sigma} \tau^{x-1}(\sigma-\tau)^{y-1}e^{-\sigma}d\tau d\sigma$
$= \int_{\sigma=0}^{\infty}\int_{\tau=0}^{\sigma} \tau^{x-1}\sigma^{y-1}(1-\frac{\tau}{\sigma})^{y-1}e^{-\sigma}d\tau d\sigma$

Again, now the comparison to $\beta(x,y)$
leads us to :
$r = \frac{\tau}{\sigma}, q = \sigma$
where the Jacobian is now:$|J| = q$.

This leads to an easy identification with the expected result:
$\Gamma(x)\Gamma(y) = \int_{q=0}^{\infty}\int_{r=0}^{1} q\,(rq)^{x-1}\,q^{(y-1)}\,(1-r)^{y-1}\,e^{-q}drdq$
$= \int_{q=0}^{\infty}\int_{r=0}^{1} r^{x-1}\,(1-r)^{y-1}\,q^{x+y-1}e^{-q}drdq$
$= \int_0^\infty q^{x+y-1}e^{-q}dq \int_0^1 r^{x-1}\,(1-r)^{y-1}\,dr$
$= \Gamma(x+y)\beta(x,y)$

As the gamma function is defined as an integral, the beta function can similarly be defined in the integral form:
$\beta_a(x) = \int_0^1 t^{a-1}(1-t)^{x-1}dt.$



Graph of Gamma Function

The trignometric form of Beta function is

$\beta(x,y) = 2\int_0^{\frac{\pi}{2}} \sin^{2x-1}\theta\cos^{2y-1}\theta d\theta,$
R(x)>0, R(y)>0.

Putting it in a form which can be used to develop integral representations of the Bessel functions and hypergeometric function,

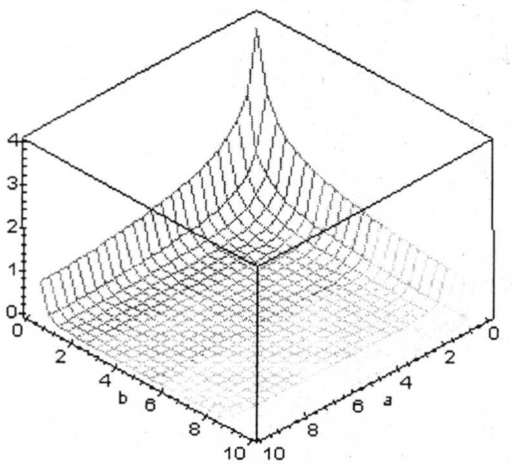
$\beta(x,y) = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}dt,$
R(x)>0, R(y)>0.

$\beta(x,y) = \frac{1}{y}\sum_{n=0}^{\infty}(-1)^n\frac{(y)_{n+1}}{n!(x+n)}$where $\Gamma(x)$

is the gamma function .The second identity shows in particular $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Just as the gamma function for integers describes factorials, the beta function can define binomial coefficient after adjusting indices.

$\binom{n}{k} = \frac{1}{(n+1)\beta(n-k+1,k+1)}$
[2][5]



Graph of the Beta Function

2 Applications:-

2.1 *Beta function and String Theory:-

The Beta function was the first known Scattering amplitude in String theory, first conjectured by Gabriele Veneziano ,an Italian theoretical physicist and a founder of string theory.

Gabriele Veneziano, a research fellow at CERN (a European particle accelerator lab) in 1968, observed a strange coincidence - many properties of the strong nuclear force are perfectly described by the Euler beta-function, an obscure formula devised for purely mathematical reasons two hundred years earlier by Leonhard Euler. In the flurry of research that followed, Yoichiro Nambu of the University of Chicago, Holger Nielsen of the Niels Bohr Institute, and Leonard Susskind of Stanford University revealed that the nuclear interactions of elementary particles modeled as one-dimensional strings instead of zero-dimensional particles were described exactly by the Euler beta-function. This was, in effect, the birth of string theory.

The Euler Beta function appeared in elementary particle physics as a model for the scattering amplitude in the so-called "dual resonance model". Introduced by Veneziano in the 1970th in order to fit experimental data , it soon turned out that the basic physics behind this model is the string(instead of the zero-dimensional mass-point).[4]

2.2 * *Preferential Attachment process:-*

Preferential Attachment to a class of processes in which some quantity, typically some form of wealth or credit, is distributed among a number of individuals or objects according to how much they already have, so that those who are already wealthy receive more than those who are not. The principal reason for scientific interest in preferential attachment is that it can, under suitable circumstances, generate power law distributions of wealth[2].[3]

2.2.1 *Stochastic Urn Process and Beta Function:-*

A preferential attachment process is a stochastic urn process, meaning a process in which discrete units of wealth, usually called "balls", are added in a random or partly random fashion to a set of objects or containers, usually called "urns". A preferential attachment process is an urn process in which additional balls are added continuously to the system and are distributed among the urns as an increasing function of the number of balls the urns already have. In the most commonly studied examples, the number of urns also increases continuously, although this is not a necessary condition for preferential attachment and examples have been studied with constant or even decreasing numbers of urns.

$P(k) = \frac{B(k+a,\gamma)}{B(k_0+a,\gamma-1)}$

Linear preferential attachment processes in which the number of urns increases are known to produce a distribution of balls over the urns following the so-called 'Yule distribution'. In the most general form of the process, balls are added to the system at an overall rate of m new species for each new urn. Each

Figure 1: Probability density Function

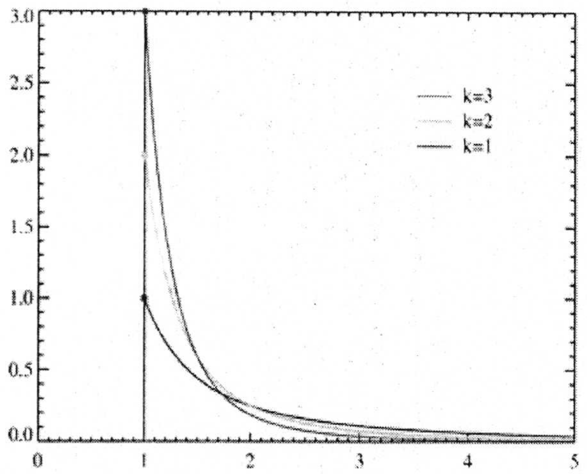


Figure 2: Pareto probability density functions for various k with $x_m = 1$.The horizontal axis is the x parameter.

newly created urn starts out with k_0 balls and further balls are added to urns at a rate proportional to the number k that they already have plus a constant $a > -k_0$. With these definitions, the fraction $P(k)$ of urns having k balls in the limit of long time is given by

$P(k) = \frac{B(k+a,\gamma)}{B(k_0+a,\gamma-1)}$

for $k \geq 0$(and zero otherwise), where $\beta(x,y)$ is the Euler beta function:

$\beta(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

with $\Gamma(x)$ being the standard gamma function, and

$\gamma = 2 + \frac{k_{0+}a}{m}$

In other words, the preferential attachment process generates a "long-tailed" distribution following a 'Pareto distribution' or 'power law' in its tail. This is the primary reason for the historical interest in preferential attachment. The species distribution and many other phenomena are observed empirically to follow power laws and the preferential attachment process is a leading candidate mechanism to explain this behavior. Preferential attachment is considered a possible candidate for, among other things, the distribution of the sizes of cities, the wealth of extremely wealthy individuals, the number of citations received by learned publications and the number of links to pages on the world wide web.[2][3]

References:-

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ON THE GAMMA FUNCTION AND ITS APPLICATIONS

JOEL AZOSE

1. Introduction

The common method for determining the value of $n!$ is naturally recursive, found by multiplying $1 * 2 * 3 * ... * (n-2) * (n-1) * n$, though this is terribly inefficient for large n. So, in the early 18th century, the question was posed: As the definition for the nth triangle number can be explicitly found, is there an explicit way to determine the value of $n!$ which uses elementary algebraic operations? In 1729, Euler proved no such way exists, though he posited an integral formula for $n!$. Later, Legendre would change the notation of Euler's original formula into that of the gamma function that we use today [1].

While the gamma function's original intent was to model and interpolate the factorial function, mathematicians and geometers have discovered and developed many other interesting applications. In this paper, I plan to examine two of those applications. The first involves a formula for the n-dimensional ball with radius r. A consequence of this formula is that it drastically simplifies the discussion of which fits better: the n-ball in the n-cube or the n-cube in the n-ball. The second application is creating the psi and polygamma functions, which will be described in more depth later, and allow for an alternate method of computing infinite sums of rational functions.

Let us begin with a few definitions: The **gamma function** is defined for $\{z \in \mathbb{C}, z \neq 0, -1, -2, \ldots\}$ to be:

$$\Gamma(z) = \int_0^\infty s^{z-1} e^{-s} ds \tag{1.1}$$

Remember some important characteristics of the gamma function:
1) For $z \in \{\mathbb{N} \setminus 0\}, \Gamma(z) = z!$
2) $\Gamma(z+1) = z\Gamma(z)$
3) $\ln(\Gamma(z))$ is convex.

The **beta function** is defined for $\{x, y \in \mathbb{C}, Re(x) > 0, Re(y) > 0\}$ to be:

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt. \tag{1.2}$$

Another identity yields:

$$B(x,y) = 2\int_0^{\pi/2} sin^{2x-1}\theta\, cos^{2y-1}\theta d\theta \tag{1.3}$$

Additionally,

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \tag{1.4}$$

A thorough proof of this last identity appears in Folland's *Advanced Calculus* [2] on pages 345 and 346. To summarize, the argument relies primarily on manipulation of $\Gamma(x)$ and $\Gamma(y)$ in their integral forms (1.1), converting to polar coordinates, and separating the double integral. This identity will be particularly important in our derivation for the formula for the volume of the n-dimensional ball later in the paper.

With these identities in our toolkit, let us begin.

2. Balls And The Gamma Function

2.1. Volume Of The N-Dimensional Ball. In his article, *The Largest Unit Ball in Any Euclidean Space*, Jeffrey Nunemacher lays down the basis for one interesting application of the gamma function, though he never explicitly uses the gamma function [3]. He first defines the open ball of radius r of dimenision n, $B_n(r)$, to be the set of points such that, for $1 \leq j \leq n$,

$$\sum x_j^2 < r^2. \tag{2.1}$$

Its volume will be referred to as $\boldsymbol{V_n(r)}$. In an argument that he describes as being "accessible to a multivariable calculus class", Nunemacher uses iterated integrals to derive his formula. He notes that, by definition:

$$V_n(r) = \underset{B_n(r)}{\iint \cdots \int} 1\, dx_1\, dx_2\, \ldots\, dx_n \tag{2.2}$$

By applying (2.1) to the limits of the iterated integral in (2.2) and performing trigonometric substitutions, he gets the following - more relevant - identity, specific to the unit ball, where $r = 1$:

(2.3) $$V_n = 2V_{n-1} \int_0^{\pi/2} cos^n\theta \, d\theta$$

In the rest of *The Largest Unit Ball in Any Euclidean Space*, Nunemacher goes on to determine which unit ball in Euclidean space is the largest. (He ultimately shows that the unit ball of dimension $n = 5$ has the greatest volume, and that the unit ball of dimension $n = 7$ has the greatest surface area, as well as - curiously - noting that V_n goes to 0 as n gets large. While a surprising result, it is not immediately relevant to the topics which I aim to pursue here. If interested, I would refer the reader to Nunemacher's article directly.) Notice, however, that this formula does not use the gamma function. We begin the derivation from here of the Gamma function form.

2.2. **Derivation.** In his 1964 article, *On Round Pegs In Square Holes And Square Pegs In Round Holes* [4], David Singmaster uses the following formula for the volume of an n-dimensional ball:

(2.4) $$V_n(r) = \frac{\pi^{n/2} r^n}{\Gamma(n/2+1)}$$

However, he never shows the derivation of this formula, and other references to Singmaster's article claim that the derivation appears explicitly in Nunemacher's article. I feel this to be an important omission, and I have endeavored here to recreate the derivation for the sake of completeness. We shall begin where Nunemacher left off with equation (2.3).

Recall (1.3) and notice its similarity to (2.3). It quickly becomes apparent that (2.3) may be rewritten as:

(2.5) $$V_n(1) = V_{n-1}(1)\, B(\frac{1}{2}, \frac{n}{2} + \frac{1}{2})$$

Continuing the recursion, we note:

(2.6) $$V_{n-1}(1) = V_{n-2}(1)\, B(\frac{1}{2}, \frac{n}{2})$$

Consequently,

(2.7) $$V_n(1) = V_1(1)\, B(\frac{1}{2}, \frac{3}{2}) \ldots B(\frac{1}{2}, \frac{n}{2})\, B(\frac{1}{2}, \frac{n}{2} + \frac{1}{2})$$

where $V_1(1) = 2 = B(\frac{1}{2}, 1)$. Substituting Gamma for Beta using (1.4) gives:

$$V_n(1) = \left[\frac{\Gamma(\frac{1}{2})\Gamma(1)}{\Gamma(\frac{3}{2})} \frac{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2})}{\Gamma(2)} \cdots \frac{\Gamma(\frac{1}{2})\Gamma(\frac{n}{2})}{\Gamma(\frac{n+1}{2})} \frac{\Gamma(\frac{1}{2})\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2}+1)} \right], \tag{2.8}$$

which telescopes to:

$$V_n(1) = \left[\frac{(\Gamma(\frac{1}{2}))^n \Gamma(1)}{\Gamma(n/2+1)} \right] \tag{2.9}$$

Since $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ and $\Gamma(1) = 1$,

$$V_n(1) = \frac{\pi^{n/2}}{\Gamma(n/2+1)} \tag{2.10}$$

Now the heavy lifting is done. Consider again the recursion relation that we used in (2.3). This recursion relation holds true for the unit ball - that is, when $r = 1$. However, when $r = 1$, we do not see the r in this equation. Instead, when we take the more general form, we get the modified recursion relation:

$$V_n = 2rV_{n-1} \int_0^{\pi/2} cos^n\theta \, d\theta \tag{2.11}$$

Going through the derivation will be virtually identical, except we have dilated the ball's size by a factor of r, and its volume by a factor of r^n. This finally yields:

$$V_n(r) = \frac{\pi^{n/2} r^n}{\Gamma(n/2+1)}, \tag{2.12}$$

which is consistent with with our original statement of (2.4). Now the derivation of the n-ball's volume using the gamma function is complete, and we may proceed to an interesting application.

2.3. **The Packing Problem.** In the motivation for his article, Singmaster explains the purpose of his article: "Some time ago, the following problem occurred to me: which fits better, a round peg in a square hole or a square peg in a round hole? This can easily be solved once one arrives at the following mathematical formulation of the problem. Which is larger: the ratio of the area of a circle to the area of the circumscribed square or the ratio of the area of a square to the area of the circumscribed circle?" [4]

The formula that we derived in the last section will prove invaluable in finding this. Since he is focusing on ratios, Singmaster uses the unit ball in both cases, though it would work similarly with any paired radius.

For the unit ball, the edge of the circumscribed cube is necessarily length 2, since it is equal in length to a diameter of the unit ball. The edge of the n-cube inscribed in the unit n-ball has length $2/\sqrt{n}$, since the diagonal of an n-cube is $\sqrt{n}$ times its edge. (Remember that the diagonal of the n-cube inscribed in the unit n-ball is the diameter of the n-ball.)

So, we construct formulas for the volume of the relevant balls and cubes using (2.4) and the facts which we have just stated:

$$V(n) = \frac{\pi^{n/2}}{\Gamma(n/2+1)}, \tag{2.13}$$

$$V_c(n) = 2^n, \tag{2.14}$$

$$V_i(n) = \frac{2^n}{n^{n/2}}, \tag{2.15}$$

where $V(n)$ represents the volume of the unit n-ball (as derived), $V_c(n)$ the volume of the circumscribed cube, and $V_i(n)$ the volume of the inscribed cube. We consider now the ratios of (2.13) to (2.14) - that is, a round peg in a square hole - and that of (2.15) to (2.13) - a square peg in a round hole.

$$R_1(n) = \frac{V(n)}{V_c(n)} = \frac{\pi^{n/2}}{2^n\Gamma\left(\frac{n+2}{2}\right)} \tag{2.16}$$

$$R_2(n) = \frac{V_i(n)}{V(n)} = \frac{2^n\Gamma\left(\frac{n+2}{2}\right)}{n^{n/2}\pi^{n/2}} \tag{2.17}$$

He then takes $\dfrac{R_1(n)}{R_2(n)}$ and applies Stirling's approximation for the gamma function: For z large,

$$\Gamma(z) \sim z^{z-1/2}\, e^{-z}\, \sqrt{2\pi} \tag{2.18}$$

Singmaster shows that as n goes to infinity, this ratio goes to zero. So, for large enough n, $R_2(n)$ is greater. By simple numerical evaluation, he determines the tipping point to be when $n = 9$. The most important result of this article is the following theorem:

Theorem. *The n-ball fits better in the n-cube better than the n-cube fits in the n-ball if and only if* $n \leq 8$.

3. Psi And Polygamma Functions

In addition to the earlier, more frequently used definitions for the gamma function, Weierstrass proposed the following:

$$\frac{1}{\Gamma(z)} = z\, e^{\gamma z} \prod_{n=1}^{\infty} (1 + z/n)\, e^{-z/n}, \tag{3.1}$$

where γ is the Euler-Mascheroni constant. Van der Laan and Temme reference another proof of this by Hochstadt [1]. This will be useful in developing the new gamma-related functions in the subsections to follow, as well as important identities. Ultimately, we will provide definitions for the **psi function** - also known as the **digamma function** - as well as the **polygamma functions**. We will then examine how the psi function proves to be useful in the computation of infinite rational sums.

3.1. **Definitions.** Traditionally, $\psi(z)$ is defined to be the derivative of $\ln(\Gamma(z))$ with respect to z, also denoted as $\dfrac{\Gamma'(z)}{\Gamma(z)}$. Just as with the gamma function, $\psi(z)$ is defined for $\{z \in \mathbb{C}, z \neq 0, -1, -2, \dots\}$. Van der Laan and Temme provide several very useful definitions for the psi function. The most well-known representation, derived from (3.1) and the definition of $\psi(z)$, is as follows:

$$\psi(z) = -\gamma - \frac{1}{z} + \sum_{n=1}^{\infty} \frac{z}{n(z+n)}, \tag{3.2}$$

though the one that we will ultimately use in the following subsection to compute sums is defined thusly:

$$\psi(z) = -\gamma - \int_0^1 \frac{t^{z-1} - 1}{1 - t}\, dt \tag{3.3}$$

This integral holds true for $Re(z) > -1$, and can be verified by expanding the denominator of the integrand and comparing to (3.2). These two are the most important definitions for the psi function, and they are the two that we will primarily use.

We will now define the **polygamma functions**, $\psi^{(k)}$.This is a family of functions stemming from the gamma and digamma functions. They are useful because they lead to better- and better-converging series. As you might imagine from the notation, the polygamma functions are the higher-order derivatives of $\psi(z)$. Consider these examples from repeated differentiation of (3.2):

$$\psi'(z) = \sum_{n=0}^{\infty} (z+n)^{-2}, \quad \psi^{(k)}(z) = (-1)^{k+1} k! \sum_{n=0}^{\infty} (z+n)^{-k-1} \tag{3.4}$$

Again, we note that, as k increases, $\psi^{(k)}(z)$ becomes more and more convergent. Now, though, we will set aside the polygamma functions and turn our focus back to the psi function and its utility in determining infinite sums.

3.2. **Use In The Computation Of Infinite Sums.** Late in their chapter on some analytical applications of the gamma, digamma, and polygamma functions, van der Laan and Temme state: "An infinite series whose general term is a rational function in the index may always be reduced to a finite series of psi and polygamma functions" [1].

Let us consider the following specific problem to motivate more general results given at the end of this section.

$$\text{Evaluate} \sum_{n=1}^{\infty} \frac{1}{(n+1)(3n+1)}. \tag{3.5}$$

We begin by expressing the summand as u_n, noting that $u_n = \dfrac{1}{3}\left(\dfrac{1}{(n+1)(n+1/3)}\right)$.

Then we perform partial fraction decomposition to yield that $\dfrac{1}{(n+1)(n+1/3)} =$

$\dfrac{3/2}{n+1} - \dfrac{3/2}{n+1/3}$, so $u_n = \dfrac{1}{2}\left(\dfrac{1}{n+1} - \dfrac{1}{n+1/3}\right)$. Remember the identity that, for all $A > 0$,

$$\frac{1}{A} = \int_0^\infty e^{-Ax}dx \tag{3.6}$$

This identity can be applied, since both denominators of both fractions are necessarily greater than 0. So the sum in (3.5) can be rewritten as:

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{(n+1)(3n+1)} &= \frac{1}{2}\sum_{n=1}^{\infty}\left[\int_0^\infty e^{-(n+1)x}dx - \int_0^\infty e^{-(n+1/3)x}dx\right] \\
&= \frac{1}{2}\sum_{n=1}^{\infty}\left[\int_0^\infty e^{-nx}e^{-x}dx - \int_0^\infty e^{-nx}e^{-(1/3)x}dx\right] \\
&= \frac{1}{2}\sum_{n=1}^{\infty}\left[\int_0^\infty e^{-nx}(e^{-x} - e^{-(1/3)x})dx\right] \\
&= \frac{1}{2}\lim_{N\to\infty}\left(\sum_{n=1}^{N}\left[\int_0^\infty e^{-nx}(e^{-x} - e^{-(1/3)x})dx\right]\right)
\end{aligned}$$

Remember from the study of infinite series that $\sum_{n=0}^{N} x^n = \dfrac{1-x^{N+1}}{1-x}$. When we subtract the first term of the series, $x^0 = 1$, we get the following result:

$$\sum_{n=1}^{N} x^n = \frac{x(1-x^N)}{1-x}. \tag{3.7}$$

Plugging in e^{-x} for x, we see:

$$\sum_{n=1}^{N} e^{-nx} = \frac{e^{-x}(1-e^{-Nx})}{1-e^{-x}} \tag{3.8}$$

Consider the relevant summation. Due to appropriate convergences following from the monotone convergence theorem, we can interchange the summation and integration and continue our manipulations of the sum.

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(3n+1)} = \frac{1}{2} \lim_{N\to\infty} \left[\int_0^{\infty} \frac{e^{-x}(1-e^{-Nx})}{1-e^{-x}} (e^{-x} - e^{-(1/3)x}) dx \right]$$

$$= \frac{1}{2} \int_0^{\infty} \frac{e^{-x}(e^{-x} - e^{-(1/3)x})}{1-e^{-x}} dx$$

Now we make use of a change of variables. Let $t = e^{-x}$. Consequently, $-e^{-x}dx = dt$. We will make this substitution. The negative sign due to this change of variable cancels with the one created by switching the limits of the integral, to yield the following:

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(3n+1)} = \frac{1}{2} \int_0^1 \frac{t - t^{1/3}}{1-t} dt$$

$$= \frac{1}{2} \int_0^1 \frac{(t-1) - (t^{1/3} - 1)}{1-t} dt$$

$$= \frac{1}{2} \int_0^1 \frac{t-1}{1-t} dt - \frac{1}{2} \int_0^1 \frac{t^{1/3} - 1}{1-t} dt$$

Compare the two integrals on the right hand side of the above equation to the formula for $\psi(z)$ in (3.3). It becomes obvious that the substitution can be made with the psi function to yield our final result:

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(3n+1)} = \frac{1}{2}\psi(4/3) - \frac{1}{2}\psi(2). \tag{3.9}$$

Professor Efthimiou of Tel Aviv University puts forth a theorem regarding series of the form

$$S(a,b) = \sum_{n=1}^{\infty} \frac{1}{(n+a)(n+b)}, \tag{3.10}$$

where $a \neq b$, and $\{a, b \in \mathbb{C}; Re(a), Re(b) > 0\}$ that generalizes the result which we have shown for a specific example above:

Theorem. $S(a,b) = \dfrac{\psi(b+1) - \psi(a+1)}{b-a}$. [5]

Let it be noted that, at present, our utility of psi functions in the calculation of infinite sums is relegated to strictly positive fractions. (Admittedly, even this is handy in a pinch, though it is hardly ideal.) However, I hope that the thorough calculation of this example is proof enough for the reader that this derivation can be made, and that the same argument could be made for a similar - that is, strictly positive - function with a denominator of degree 2. If a doubt persists, I urge the reader to create a rational function of this form and follow the same steps as my proof to derive an equivalence with a sum of psi and/or polygamma functions.

4. Future Works

Van der Laan and Temme propose that *every* infinite series of rational functions may be reduced to a finite series of psi and polygamma functions. This seems plausible, but the statement requires more rigorous examination to be taken as sound. The subjects that I would like to delve the most deeply into are what I touched on at the very end with Prof. Efthimiou's theorem and the limits on the utility of the psi function in the calculation of infinite sums. I think that it would be a worthwhile endeavor to try to formulate an analogue of Efthimiou's theorem for a function with denominator of degree n. Finally, I would like to work on examining what could be done with infinite sums of fractions that are not strictly positive. I would like to determine if there is a similar formula for these series, as well.

5. Conclusion

In the first section of this paper, we provided definitions for the gamma function. We then went through a gamma derivation for the formula of the volume of an n-ball and used that in working with ratios involving inscribed and circumscribed cubes to determine the following:

Theorem. *The n-ball fits better in the n-cube better than the n-cube fits in the n-ball if and only if* $n \leq 8$.

In the second section, we presented the psi - also known as the digamma - function and the family of polygamma functions. We expressed a specific infinite sum as the finite sum of psi functions as motivation for the following more general result:

Theorem. *For* $a \neq b$, *and* $\{a, b \in \mathbb{C}; Re(a), Re(b) > 0\}$, $\sum_{n=1}^{\infty} \dfrac{1}{(n+a)(n+b)} = \dfrac{\psi(b+1) - \psi(a+1)}{b-a}$.

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WEB
REFERENCE

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COURSE – PLAN – One copy to be submitted to the HOD one week before commencement of the semester

Subject Code	Name of the Subject	Class/Sem	Name of the Faculty / Designation	Number of Students	Total Proposed Periods per semester/year	
					Lectures	Tutorial
MA202BS	Mathematics II	IT / II SEM.	Dr. T.V.A.P.SASTRY	60	99	9

WEEK Number	Lecture Number	Topic	Web references
Wk1	1	**UNIT-I Laplace transforms** Introduction Laplace transforms of standard functions, linearity property	http://lpsa.swarthmore.edu/ LaplaceXform/FwdLaplace/LaplaceProps.html
	2	Problems of Laplace transform	http://tutorial.math.lamar.edu /Classes/DE/LaplaceTransforms.aspx
	3	First shifting theorem and its problems, unit step function	http://17calculus.com/laplace-transforms/shifting-theorems/
	4	Second shifting theorem , change of scale property and their problems	http://www.mathalino.com/reviewer/advance-engineering-mathematics/change-scale-property-laplace-transform
	7	Laplace transforms of derivatives , integrals and their problems	http://mathfaculty.fullerton.edu /mathews/c2003/LaplaceDiffIntegrateMod.html
	8	TUTORIAL 1	http://lpsa.swarthmore.edu /LaplaceXform/FwdLaplace/LaplaceProps.html
	9	Laplace transforms multiplication with t and	http://mathfaculty.fullerton.edu /mathews/c2003/LaplaceMultDivMod.html

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		related problems	
Wk 2	10	Laplace transforms of division with t and related problems	http://www.mathalino.com/reviewer/advance-engineering-mathematics/division-t-laplace-transform
	11	Dirac delata function , and its laplace transform, periodic function and its laplace transform and related problems	http://tutorial.math.lamar.edu/Classes/DE/DiracDeltaFunction.aspx
	12	Problems on evaluation of integrals by laplace transform	https://math.stackexchange.com /questions/243800/integrating-using-laplace-transforms
	13	Revision of periodic functions and evaltuation of integrals.	http://www.askiitians.com/revision-notes/maths/indefinite-integral/
	14	Inverse Laplace transforms and basic formulae, few problems	http://tutorial.math.lamar.edu/Classes/DE/InverseTransforms.aspx
	15	Problems on inverse laplace transforms .	http://tutorial.math.lamar.edu/Classes/DE/InverseTransforms.aspx
	16	First shifting, second shifting, change of scale and related problems	http://www.mathalino.com/reviewer/advance-engineering-mathematics/first-shifting-property-laplace-transform
	17	Revision of all the above topic of inverse laplace transform.	http://tutorial.math.lamar.edu/Classes/DE/InverseTransforms.aspx
	18	Inverse laplace transform of derivatives and integrals and	http://www.math.fsu.edu/~fusaro/EngMath/Ch5/LTIF.html

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Wk 3		related problems	
	18	Inverse laplace transform of mult and division with s and related problems	http://www.mathalino.com/reviewer/advance-engineering-mathematics/division-t-laplace-transform
	19	Problems on all the above 4 topics	http://www.mathalino.com/reviewer/advance-engineering-mathematics/division-t-laplace-transform
Wk 4	20	Convolution theorem and related problems	http://mathfaculty.fullerton.edu/mathews/c2003/laplaceconvolutionmod.html
	21	Problems related to Convolution theorem	http://mathfaculty.fullerton.edu/mathews/c2003/laplaceconvolutionmod.html
	22	Solving ODE by Laplace transforms	http://tutorial.math.lamar.edu/Classes/DE/IVPWithLaplace.aspx
	23	Solving ODE by Laplace transforms	http://tutorial.math.lamar.edu/Classes/DE/IVPWithLaplace.aspx
	24	TUTORIAL 2	https://www.quora.com/What-are-the-real-world-applications-of-Laplace-transform-especially-in-computer-science
	25	UNIT-II Introduction to improper integrals and beta functions	https://mathematicsresources.files.wordpress.com/2015/02/l-betaandgammafunctions.pdf
Wk5	26	Properties of Beta function and evaluation	https://en.wikipedia.org/wiki/Beta_function
	27	Forms of beta function	http://mathworld.wolfram.com/BetaFunction.html
	28	Forms of beta function	http://mathworld.wolfram.com/BetaFunction.html
	29	Solving problems on beta functions	http://mathworld.wolfram.com/BetaFunction.html
	30	Solving problems on beta functions	http://mathworld.wolfram.com/BetaFunction.html

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	31	Definition of gamma function, few formulae, Properties of Gamma function	http://functions.wolfram.com/GammaBetaErf /Gamma/introductions/Gamma/ShowAll.html
	32	beta gamma relation , few problems	http://math.tutorvista.com/statistics/beta-function.html
	33	Evaluation of integrals using Beta and Gamma functions	https://math.stackexchange.com/questions/834654/evaluating-integral-using-beta-and-gamma-functions
	34	Evaluation of integrals using Beta and Gamma functions	https://math.stackexchange.com/questions/834654/evaluating-integral-using-beta-and-gamma-functions
	35	Evaluation of integrals using Beta and Gamma functions	https://math.stackexchange.com/questions/834654/evaluating-integral-using-beta-and-gamma-functions
	36	Evaluation of integrals using Beta and Gamma functions	https://math.stackexchange.com/questions/834654/evaluating-integral-using-beta-and-gamma-functions
	37	TUTORIAL 3	http://www.math.wvu.edu/~hjlai/Teaching/Tip-Pdf/Tip1-30.pdf
	38	Revision	https://math.stackexchange.com/questions/834654/evaluating-integral-using-beta-and-gamma-functions
Wk7	39	Evaluation of integrals using Beta and Gamma functions	https://math.stackexchange.com/questions/834654/evaluating-integral-using-beta-and-gamma-functions
	40	UNIT-III Introduction to Multiple integrals, Double integrals in Cartesian coordinates	http://www.personal.soton.ac.uk/jav/soton/HELM/workbooks/ workbook_27/27_2_mult_ints_non_rec_region.pdf
	41	Evaluation of double integrals in Cartesian	http://tutorial.math.lamar.edu/Classes/CalcIII/DIPolarCoords.aspx

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		coordinates	
	42	Evaluation of double integrals in Cartesian coordinates	http://tutorial.math.lamar.edu/Classes/CalcIII/DIPolarCoords.aspx
Wk8	43	Evaluation of double integrals in polar coordinates	http://tutorial.math.lamar.edu/Classes/CalcIII/DIPolarCoords.aspx
	44	Evaluation of double integrals in polar coordinates	http://tutorial.math.lamar.edu/Classes/CalcIII/DIPolarCoords.aspx
	45	Conversion of Cartesian to polar and vice versa and related problems	http://www.analyzemath.com/polarcoordinates/polar_rectangular.html
	46	Change of order of integration method and related problems	http://mathinsight.org/double_integral_change_order_integration_examples
	47	TUTORIAL 4	http://tutorial.math.lamar.edu/Classes/CalcIII/DIPolarCoords.aspx
	48	Change of order of Integration problems	http://mathinsight.org/double_integral_change_order_integration_examples
wk 9	49	Change of order of Integration problems	http://mathinsight.org/double_integral_change_order_integration_examples
	50	Revision	http://mathinsight.org/double_integral_change_order_integration_examples
	51	Triple integrals in Cartesian formulae	https://www.math24.net/calculation-volumes-triple-integrals/
	52	Problems on triple integrals	https://www.math24.net/calculation-volumes-triple-integrals/
	53	Revision	https://www.math24.net/calculation-volumes-triple-integrals/
	54	Triple integrals in spherical and	http://tutorial.math.lamar.edu/Classes/CalcIII/TISphericalCoords.aspx

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wk 11		cylindrical formulae and problems	
	55	Evaluation of triple integrals in spherical and cylindrical	http://tutorial.math.lamar.edu/Classes/CalcIII/TICylindricalCoords.aspx
	56	Conversion of triple integrals	http://tutorial.math.lamar.edu/Classes/CalcIII/TICylindricalCoords.aspx
	57	Finding areas using double integrals	http://mathinsight.org/double_integral_area
wk 12	58	Finding volumes using double integrals	http://mathinsight.org/double_integral_volume
	59	Volume of a region using triple Integration.	http://tutorial.math.lamar.edu/Classes/CalcIII/TripleIntegrals.aspx
	60	Finding the center of gravity using Beta and Gamma functions	https://math.stackexchange.com/questions/1138347 /calculate-integral-using-beta-and-gamma-functions
	61	Revision	https://math.stackexchange.com/questions/1138347 /calculate-integral-using-beta-and-gamma-functions
	62	UNIT-IV **Vector differentiation**: introduction to vectors , few defns like gradient, curl, and divergence	http://www.maths.tcd.ie/~evd/2E2/Notes/vector,grad,div,curl.pdf
	63	Gradient of a Scalar point function, few formulae and related problems	Gradient of a Scalar point function, few formulae and related problems
	64	Problems on above topic	Gradient of a Scalar point function, few formulae and related problems
wk 13	65	Directional derivative problems	http://tutorial.math.lamar.edu/Classes/CalcIII/DirectionalDeriv.aspx
	66	Angle between normals	http://www.vitutor.com/geometry/distance/angle_planes.html

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	67	Divergence of vector point function and few problems	https://en.wikipedia.org/wiki/Divergence
	68	Solinoidal problems and problems related to divergence	https://en.wikipedia.org/wiki/Solenoidal_vector_field
	69	Curl of a vector function and related problems	https://en.wikipedia.org/wiki/Curl_(mathematics)
	70	Irrotational of a vector function and other problems	http://mathworld.wolfram.com/IrrotationalField.html
wk 14	71	Laplacian operator and related problems	https://www.plymouth.ac.uk/uploads/production /document/path/3/3744/PlymouthUniversity_MathsandStats_the_Laplacian.pdf
	72	Vector identities	https://en.wikipedia.org/wiki/Vector_calculus_identities
	73	Problems on Vector identities	https://www.cse.iitb.ac.in/~cs749/spr2017/handouts/jem_graddivcurl.pdf
	74	TUTORIAL 5	http://academic.brcc.edu/ryanl/modules/multivariable /integration/applications/applications_fi.pdf
wk 15	75	UNIT-V Vector Integration Introduction, line integrals and related formulae	http://tutorial.math.lamar.edu/Classes/CalcIII/LineIntegralsVectorFields.aspx
	76	Line integral – work done problems	https://www.khanacademy.org/math/multivariable-calculus/integrating-multivariable-functions/line-integrals-vectors/v/using-a-line-integral-to-find-the-work-done-by-a-vector-field-example
	77	Problems on line integrals	http://tutorial.math.lamar.edu/Classes/CalcIII/LineIntegralsPtI.aspx
	78	Problems on line integrals	http://tutorial.math.lamar.edu/Classes/CalcIII/LineIntegralsPtI.aspx

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	79	Surface integrals and related formulae and problems	http://tutorial.math.lamar.edu/Classes/CalcIII/SurfaceIntegrals.aspx
	80	Problems on Surface integrals	http://tutorial.math.lamar.edu/Classes/CalcIII/SurfaceIntegrals.aspx
wk 16	81	Problems on Surface integrals	http://tutorial.math.lamar.edu/Classes/CalcIII/SurfaceIntegrals.aspx
	82	Volume integrals and related formulae	http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx
	83	Problems on Volume integrals	http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx
	84	Problems on Volume integrals	http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx
	85	Vector integral theorems introduction and Greens theorem statement	http://www.personal.soton.ac.uk/jav/soton/HELM /workbooks/workbook_29/29_3_int_vec_thms.pdf
	86	TUTORIAL 6	https://math.dartmouth.edu/archive/m11f10/public_html/exams/m11f08-exam1practice-solutions.pdf
	87	Problems on Greens theorem	http://tutorial.math.lamar.edu/Classes/CalcIII/GreensTheorem.aspx
wk 17	88	Problems on greens theorem	http://tutorial.math.lamar.edu/Classes/CalcIII/GreensTheorem.aspx
	89	Revision	http://tutorial.math.lamar.edu/Classes/CalcIII/GreensTheorem.aspx
	90	Stroke theorem and related problems	http://tutorial.math.lamar.edu/Classes/CalcIII/StokesTheorem.aspx
	91	Problems on Stroke theorem	http://tutorial.math.lamar.edu/Classes/CalcIII/StokesTheorem.aspx
	92	Problems on strokes theorem	http://tutorial.math.lamar.edu/Classes/CalcIII/StokesTheorem.aspx
	93	Gauss divergence theorem statement and related	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf

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		problems	
wk 18	94	Problems on Gauss divergence theorem	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf
	95	Problems on guass divergence theorem	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf
	96	revision	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf
	97	revision	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf
	98	revision	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf
	99	revision	http://www-users.math.umn.edu/~nega0024/docs/2263_S14/GaussExamples.pdf

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LECTURE NOTES

1) $\bar{a}$

→ Prove that

$\nabla f \times \nabla g$ is Solenoidal

We Know that

~~Rotl a =~~

$\nabla\cdot(\bar{a}\times\bar{b}) = \bar{b}\cdot(\nabla\times\bar{a}) - \bar{a}\cdot(\nabla\times\bar{b})$.

Put $\bar{a} = \nabla f$, $\bar{b} = \nabla g$.

L.H.S :

$$\nabla\cdot(\nabla f\times\nabla g).$$

$= \nabla g\cdot(\nabla\times\nabla f) - \nabla f\cdot(\nabla\times\nabla g)$

$= \nabla g\cdot$ Curl grad f $- \nabla f\cdot$ Curl grad g

$= \nabla g\cdot\bar{0} - \nabla f\cdot\bar{0}$

$= 0.$

$\nabla\cdot(\nabla f\times\nabla g) = 0.$

$\text{div}(\nabla f\times\nabla g) = 0.$

→ If f and g are two scalar point functions then prove that $\text{div}(f\nabla g) = f\nabla^2 g + \nabla f\cdot\nabla g$

$$\text{div}(f\nabla g) = f\nabla^2 g + \nabla f\cdot\nabla g$$

$$\nabla g = i\frac{\partial g}{\partial x} + j\frac{\partial g}{\partial y} + k\frac{\partial g}{\partial z}$$

$$f\nabla g = i f\frac{\partial g}{\partial x} + j f\frac{\partial g}{\partial y} + k f\frac{\partial g}{\partial z}$$

$$\operatorname{div}(f\nabla g) = \nabla\cdot(f\nabla g)$$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot\left(i f\frac{\partial g}{\partial x} + j f\frac{\partial g}{\partial y} + k f\frac{\partial g}{\partial z}\right).$$

$$= \frac{\partial}{\partial x}\left[f\frac{\partial g}{\partial x}\right] + \frac{\partial}{\partial y}\left[f\frac{\partial g}{\partial y}\right] + \frac{\partial}{\partial z}\left[f\frac{\partial g}{\partial z}\right]$$

$$f\left[\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}\right] + \left[\frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial x} + \frac{\partial f}{\partial y}\cdot\frac{\partial g}{\partial y} + \frac{\partial f}{\partial z}\cdot\frac{\partial g}{\partial z}\right]$$

$$= f\nabla^2 g + \left[i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}\right]\cdot\left[i\frac{\partial g}{\partial x} + j\frac{\partial g}{\partial y} + k\frac{\partial g}{\partial z}\right]$$

$$= f\nabla^2 g + \nabla f\cdot\nabla g$$

$$= \text{RHS}.$$

→ Prove that

$$\text{curl}(\bar{a}\times\bar{b}) = \bar{a}\,\text{div}\,\bar{b} - \bar{b}\,\text{div}\,\bar{a} + (\bar{b}\cdot\nabla)\bar{a} - (\bar{a}\cdot\nabla)\bar{b}$$

$$\nabla\times(\bar{a}\times\bar{b})$$

$$= \sum i\times\frac{\partial}{\partial x}(\bar{a}\times\bar{b})$$

$$= \sum i\times\left[\frac{\partial\bar{a}}{\partial x}\times\bar{b} + \bar{a}\times\frac{\partial\bar{b}}{\partial x}\right]$$

$$= \sum\left[i\times\left(\frac{\partial\bar{a}}{\partial x}\times\bar{b}\right) + i\times\left(\bar{a}\times\frac{\partial\bar{b}}{\partial x}\right)\right]$$

$$= \sum\left[i\times\left(\frac{\partial\bar{a}}{\partial x}\times\bar{b}\right)\right] + \sum\left[i\times\left(\bar{a}\times\frac{\partial\bar{b}}{\partial x}\right)\right]$$

$$= \sum\left[(i\cdot\bar{b})\frac{\partial\bar{a}}{\partial x} - \left(i\cdot\frac{\partial\bar{a}}{\partial x}\right)\bar{b}\right] + \sum\left[\left(i\cdot\frac{\partial\bar{b}}{\partial x}\right)\bar{a} - (i\cdot\bar{a})\frac{\partial\bar{b}}{\partial x}\right]$$

$$= \sum(i\cdot\bar{b})\frac{\partial\bar{a}}{\partial x} - \sum\left(i\cdot\frac{\partial\bar{a}}{\partial x}\right)\bar{b} + \sum\left(i\cdot\frac{\partial\bar{b}}{\partial x}\right)\bar{a} - \sum(i\cdot\bar{a})\frac{\partial\bar{b}}{\partial x}$$

$$\cancel{= \bar{a}\,\text{div}\,\bar{b} - \bar{b}\,\text{div}\,\bar{a} + (\bar{b}\cdot\nabla)\bar{a} - (\bar{a}\cdot\nabla)\bar{b}}$$

$$\sum\left(\bar{b}\cdot i\frac{\partial}{\partial x}\right)\bar{a} - (\nabla\cdot\bar{a})\bar{b} + (\nabla\cdot\bar{b})\bar{a} - \sum\left(\bar{a}\cdot i\frac{\partial}{\partial x}\right)\bar{b}$$

$$= (\bar{b}\cdot\nabla)\bar{a} - \bar{b}\,\text{div}\,a + \bar{a}\,\text{div}\,b - (\bar{a}\cdot\nabla)\bar{b}$$

Find $(A\cdot\nabla)\phi$ at $(1,-1,1)$

if $A = 3xyz^2 i + 2xy^3 j - x^2yz k$

and $\phi = 3x^2 - yz$

Given

$A = 3xyz^2 i + 2xy^3 j - x^2yz K$

W.K.T

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + K\frac{\partial}{\partial z}$$

$$(A\cdot\nabla) = \frac{\partial}{\partial x}(3xyz^2) + \frac{\partial}{\partial y}(2xy^3) + \frac{\partial}{\partial z}(x^2yz)$$

$(A\cdot\nabla)\phi$

$$= \left(A\cdot i\frac{\partial}{\partial x}\right)\phi + \left(A\cdot j\frac{\partial}{\partial y}\right)\phi + \left(A\cdot K\frac{\partial}{\partial z}\right)\phi$$

$$= (A\cdot i)\frac{\partial\phi}{\partial x} + (A\cdot j)\frac{\partial\phi}{\partial y} + (A\cdot K)\frac{\partial\phi}{\partial z}$$

$$= (3xyz^2)(6x) + (2xy^3)(-z) + (x^2yz)(y)$$

$$= 18x^2yz^2 - 2xy^3z + x^2y^2z$$

$$= 18(1)^2(-1)(1)^2 - 2(1)(-1)^3(1) + \cancel{x}^2(-1)^2(1)$$

$$= -18 + 2 + 1 = -15$$

$=$

UNIT-V

VECTOR INTEGRATION.

Line Integration (Single Integration) / (Workdone by force):

The process of finding Integration along the line or curve or the path is Known as line Integration

→ It is denoted by $\int_C F \cdot dr$

Cartesian Form:

$$F = F_1 i + F_2 j + F_3 k.$$

$$\bar{r} = xi + yj + zk$$

$$dr = dx\,i + dy\,j + dz\,k$$

$$F \cdot dr = F_1 dx + F_2 dy + F_3 dz.$$

$$\int_C F \cdot dr = \int_C [F_1 dx + F_2 dy + F_3 dz]$$

→ Evaluate $\int_C F \cdot dr$ where $F = 3xy\,i - y^2 j$ along the curve $y = 2x^2$ from $(0,0)$ to $(1,2)$ in xy plane.

$$F = 3xy\,i - y^2 j$$

$y = 2x^2$ from $(0,0)$ to $(1,2)$.

on XY plane, $z = 0$

$$dz = 0$$

$dr = dx\,i + dy\,j$

$F = 3xy\,i - y^2 j$

$\boxed{F \cdot dr = 3xy\,dx - y^2 dy}$

given $y = 2x^2$

$dy = 4x\,dx$

$F \cdot dr = 3x(2x^2)\,dx - (2x^2)^2 \cdot 4x\,dx.$

$= 6x^3 dx - 16x^5 dx.$

$\therefore \int_C F \cdot dr = \int [6x^3 - 16x^5)\,dx.$

$= \frac{6}{4}(x^4)_0^1 - \frac{16}{6}(x^6)_0^1$

$= \frac{3}{2} - \frac{8}{3} = \frac{9-16}{6} = -7/6.$

→ Evaluate $\int_C F \cdot dr$ where $F = (x-3y)i + (y-2x)j$ and c is the closed curve in xy plane where $x = 2\cos t$, $y = 3\sin t$ from $t=0$ to 2π

$z = 0$

$dz = 0$

$dr = dx\,i + dy\,j$

$F \cdot dr = (x-3y)\,dx + (y-2x)\,dy.$

$F \cdot dr = x\,dx - 3y\,dx + y\,dy - 2x\,dy$.

$x = 2\cos t \quad , \quad y = 3\sin t$

$dx = -2\sin t \cdot dt \qquad dy = 3\cos t\,dt$

$F \cdot dr = 2\cos t \cdot (-2\sin t\,dt) - 3(3\sin t)(-2\sin t\,dt)$

$+ \; 3\sin t\,(3\cos t\,dt) - 2\,(2\cos t)(3\cos t\,dt)$

$= -4\cos t\sin t\,dt + 18\sin^2 t\,dt + 9\sin t\cos t\,dt$

$- \; 12\cos^2 t\,dt$

$= \; 5\sin t\cos t\,dt \; 18\sin^2 t\,dt - 12\cos^2 t\,dt$

$\int_C F \cdot dr = \int_C \left(5\sin t\cos t + 18\sin^2 t - 12\cos^2 t\right) dt$

$= \int_0^{2\pi} \frac{5}{2}(2\sin t\cos t)\,dt + 18\int_0^{2\pi}\left(\frac{1-\cos 2t}{2}\right)dt - 12\int_0^{2\pi}\left(\frac{1+\cos 2t}{2}\right)dt$

$= \left[\frac{5}{2}\frac{\cos 2t}{2}\right]_0^{2\pi} + 18\left[\frac{t}{2} - \frac{\sin 2t}{4}\right]_0^{2\pi} - 12\left[\frac{t}{2} + \frac{\sin 2t}{4}\right]_0^{2\pi}$

$= -\frac{5}{4}\left((1)-1\right) + 18(\pi - 0) - 12(\pi + 0)$

$= \; 18\pi - 12\pi$

$= \; 6\pi$.

$y)i$

$.\sin t$

→ If $F = (5xy - 6x^2)i + (2y - 4x)j$

Find the Workdone by the force where c is the Curve in xy plane from (1,1) to (2,8) and $y = x^3$.

→ If $F = xyi - zj + x^2K$ where c is the curve $x = t^2$ $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$

$$r = xi + yj + zK$$

$$dr = dxi + dyj + dzK.$$

$$F \cdot dr = xydx - zdy + x^2dz$$

we Know that

$$x = t^2, \quad y = 2t, \quad z = t^3$$

$$dx = 2tdt \quad dy = 2dt, \quad dz = 3t^2dt$$

$$F \cdot dr = t^2(2t)(2tdt) - t^3(2dt) + t^4(3t^2dt)$$

$$= 4t^4dt - 2t^3dt + 3t^6dt$$

$$\int F \cdot dr = \int_0^1 dt(4t^4 - 2t^3 + 3t^6).$$

$$= 4\left[\frac{t^5}{5}\right]_0^1 - 2\left[\frac{t^4}{4}\right]_0^1 + 3\left[\frac{t^7}{7}\right]_0^1$$

$$= \frac{4}{5} - \frac{1}{2} + \frac{3}{7} = \frac{51}{70}$$

→ Find

in the f

$F = 3$

st line

$F \cdot dr =$

Given

eqn of

$=$

$x =$

$dx = 2d$

$F \cdot dr =$

$= 24t$

$= 36$

t Vari

$\int_0^1 36t^2$

→ Find the Workdone in moving a particle in the force field ~~$F = 3x^2 i + j(2xz) - (y) + z$~~

$F = 3x^2 i + j(2xz - y) + zk$ along the st line from $(0,0,0)$ to $(2,1,3)$

$r = xi + yj + zk$

$dr = dx\,i + dy\,j + dz\,k$

$F \cdot dr = 3x^2 dx + 2xz\,dy - y\,dy + z\,dz$

Given $O(0,0,0)$ $A(2,1,3)$

eqn of OA $= \dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1} = t$

$= \dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z}{3} = t$

$x = 2t$, $y = t$, $z = 3t$

$dx = 2dt$ $dy = dt$ $dz = 3dt$

$F \cdot dr = 3(2t)^2 2dt + 2(2t)(3t)dt - t\,dt + 3t(3dt)$

$= 24t^2 dt + 12t^2 dt + 8t\,dt$

$= 36t^2 dt + 8t\,dt$

t Varies from 0 to 1

$\int_0^1 36t^2 dt + 8t\,dt = \dfrac{36}{3}[t^3]_0^1 + \dfrac{8}{2}[t^2]_0^1$

$= 12 + 4$

$= 16$

Evaluate $\int_C F\cdot dr$ where $F = yz\,i + xz\,j + xy\,k$ over a helix $x = a\cos t$, $y = a\sin t$, $z = kt$ where z varies from 0 to 2π

$$F = yz\,i + xz\,j + xy\,k$$

$$dr = dx\,i + dy\,j + dz\,k.$$

$$F\cdot dr = yz\,dx + xz\,dy + xy\,dz.$$

$$x = a\cos t, \quad y = a\sin t, \quad z = kt$$

$$dx = -a\sin t\,dt, \quad dy = a\cos t\,dt, \quad dz = k\,dt$$

$$F\cdot dr = (a\sin t)(kt)(-a\sin t\,dt) + (a\cos t)(kt)(a\cos t\,dt) + (a\cos t)(a\sin t)(k\,dt)$$

$$= -a^2 t\sin^2 t\cdot k\cdot dt + k a^2\cos^2 t\cdot t\,dt + a^2\cos t\sin t\,k$$

$$\cancel{= 2k(-a t\sin^2 t + a^2 t\cos^2 t}$$

$$= k a^2(-t\sin^2 t + t\cos^2 t + \cos t\sin t)\,dt.$$

$$k a^2\int_0^{2\pi}\left(t(\cos 2t) + \frac{1}{2}\sin 2t\right)dt$$

$$k a^2\left\{\left[t\cdot\frac{\sin 2t}{2} - \int\frac{\sin 2t}{2}\,dt\right]_0^{2\pi} + \frac{1}{2}\left[\frac{\cos 2t}{2}\right]_0^{2\pi}\right\}$$

$$= k a^2\left[\left[\frac{t\sin 2t}{2} + \frac{1}{4}\cos 2t\right]_0^{2\pi} + \left[\frac{1}{4}\cos 2t\right]_0^{2\pi}\right]$$

$j + xy\,k$

$= Ka^2\left[\left(\frac{1}{4}-\frac{1}{4}\right)-\left(\frac{1}{4}-\frac{1}{4}\right)\right]$

$\to 2\pi$

$= 0.$

$\to \int_S F \cdot n\, ds$ where $F = 18zi - 12j + 3yk$ and S is the part of the surface of the plane $2x+3y+6z = 12$ located in the first Octant

$k\,dt$

given $\phi = 2x+3y+6z-12$

$)(a\cos t\, dt)$

$\frac{\partial \phi}{\partial x} = 2, \quad \frac{\partial \phi}{\partial y} = 3, \quad \frac{\partial \phi}{\partial z} = 6.$

$\cos t \sin t\, k.$

$\therefore \nabla\phi = i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{\partial y} + k\frac{\partial \phi}{\partial z}$

$= 2i + 3j + 6k.$

$|\nabla\phi| = \sqrt{4+9+36} = 7$

$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2i+3j+6k}{7}$

$\left.\frac{\cos 2t}{2}\right]_0^{2\pi}$

$F = 18zj - 12j + 3yk$

$F \cdot n = \frac{1}{7}(36z - 36 + 18y)$

$= \frac{18}{7}(2z - 2 + y)$

$]$

Let R is the projection on XY plane

$$ds = \frac{dx\,dy}{|\hat{n}\cdot K|} = \frac{dx\,dy}{\left|\frac{6}{7}\right|}$$

$$F\cdot\hat{n}\,ds = \frac{\cancel{18}^{\,3}}{\cancel{7}}(2z - x + y)\cdot\frac{dx\,dy}{\cancel{6}}\cdot\cancel{7}$$

$$= (6z - 6 + 3y)\,dx\,dy$$

$2x + 3y + 6z = 12$

$6z = 12 - 2x - 3y$

$$= (12 - 2x - \cancel{3y} - 6 + \cancel{3y})\,dx\,dy$$

$$= (6 - 2x)\,dx\,dy$$

$$\int_S F\cdot\hat{n}\,ds = \iint (6 - 2x)\,dx\,dy$$

$$= \int_{y=0}^{4}\int_{x=0}^{\frac{12-3y}{2}} 6\,dx\,dy - 2\int_{y=0}^{4}\int_{x=0}^{\frac{12-3y}{2}} x\,dx\,dy$$

$2x + 3y + 6z = 12$

on XY plane $z = 0$

$2x = 12 - 3y$

$x = \frac{12-3y}{2} = 6 - \frac{3}{2}y$

put $x = 0$ to get y limits

$12 = 3y$

$y = 4$

$144 + 9y^2 - 72y$

$$= 6\int_{y=0}^{4}[x]_0^{\frac{12-3y}{2}}\,dy - \frac{2}{2}\int_{y=0}^{4}(x^2)_0^{\frac{12-3y}{2}}\,dy$$

$$= \frac{6}{2}\int_{y=0}^{4}(12 - 3y)\,dy - \frac{1}{4}\int_{y=0}^{4}(12 - 3y)^2\,dy$$

$$= 3\left[12y - \frac{3y^2}{2}\right]_0^4 - \frac{1}{4}\left[144 + 9y^2 - 72y\right]_{y=}^{4}$$

$$3\left[48 - \frac{48}{2}\right] - \frac{1}{4}\left[144 + 144 - 288\right]$$

$$= 0$$

$$= 3\left[12y - \frac{3y^2}{2}\right]_0^4 - \frac{1}{4}\left[144y + \frac{9y^3}{3} - \frac{72y^2}{2}\right]_0^4$$

$$= 3\left[48 - 24\right] - \frac{1}{4}\left[\cancel{576} + \frac{192}{3} - \cancel{576}\right]$$

$$= 24 \times 3 - 24 \times 2$$

$$= 24$$

→ Evaluate $\int_S F \cdot n \, ds$ where

$F = 12x^2y\,i - 3yz\,j + 2z\,K$ and S is the portion of the plane $x + y + z = 1$ included in the first Octet.

$$\phi = x + y + z - 1$$

$$\frac{\partial\phi}{\partial x} = 1, \quad \frac{\partial\phi}{\partial y} = 1, \quad \frac{\partial\phi}{\partial z} = 1$$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + K\frac{\partial\phi}{\partial z}$$

$$= \hat{i} + \hat{j} + \hat{K}$$

$$|\nabla\phi| = \sqrt{1+1+1} = \sqrt{3}$$

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\hat{i} + \hat{j} + \hat{K}}{\sqrt{3}}$$

$$F = 12x^2y\,i - 3yz\,j + 2z\,k$$

$$F \cdot \hat{n} = \frac{1}{\sqrt{3}}\left[12x^2y - 3yz + 2z\right]$$

Let R is the Projection on XY plane

$$ds = \frac{dx\,dy}{|\hat{n} \cdot k|} = \frac{dx\,dy}{1/\sqrt{3}}$$

$$F \cdot \hat{n}\,ds = \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\left[12x^2y - 3yz + 2z\right] dx\,dy.$$

$$x + y + z = 1$$

$$z = 1 - x - y$$

$$= \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\left[12x^2y - 3y(1-x-y) + 2(1-x-y)\right] dx\,dy$$

$$= \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\left[12x^2y - 3y + 3xy + 3y^2 + 2 - 2x - 2y\right] dx\,dy$$

$$= \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\left[12x^2y + 3y^2 - 5y + 3xy - 2x + 2\right] dx\,dy.$$

$$\int_S F \cdot \hat{n}\,ds = \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\iint \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\left(12x^2y + 3y^2 - 5y + 3xy - 2x + 2\right) dx\,dy$$

$$x + y + z = 1$$

$$z = 0$$

$$x + y = 1$$

$$x = 1 - y$$

$$\frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\int_{y=0}^{1}\int_{x=0}^{x=1-y}\left(12x^2y + 3y^2 + 3xy - 5y - 2x + 2\right) dx\,dy.$$

$$= \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\int_0^1 12y\left[\frac{x^3}{3}\right]_0^{1-y} + 3y^2\left[x\right]_0^{1-y} + 3y\left[\frac{x^2}{2}\right]_0^{1-y} - 5y\left[x\right]_0^{1-y} - 2\left(\frac{x^2}{2}\right)_0^{1-y} + 2\left(x\right)_0^{1-y}$$

$$= \frac{1}{3}\int_0^1 \Big[\frac{12}{3}y(1-y)^3 + 3y^2(1-y) + \frac{3}{2}y(1-y)^2$$

$$- 5y(1-y) - (1-y)^2 + 2(1-y)\Big)\Big]$$

$$\Rightarrow \frac{1}{3}\int_0^1 4y\big(1+y^3-3y(1-y)\big) + 3y^2 - 3y^3 + \frac{3}{2}y(1+y^2-2y)$$

$$- 5y + 5y^2 - (1+y^2-2y) + 2 - 2y\Big]$$

$$= \frac{1}{3}\int_0^1 4y + 4y^4 - 12y^2 + 12y^3 + \cancel{3y^2} - 3y^3 + \frac{3}{2}y + \frac{3}{2}y^3$$

$$- \cancel{3y^2} - 5y + 5y^2 - 1 - y^2 + \cancel{2y} + 2 - \cancel{2y}\Big]$$

$$= \int_0^1 \frac{21}{2}y^3 + 4y^4 - 8y^2 + \frac{11}{2}y$$

$$= \frac{21}{2} + 4 - 8 + \frac{11}{2}$$

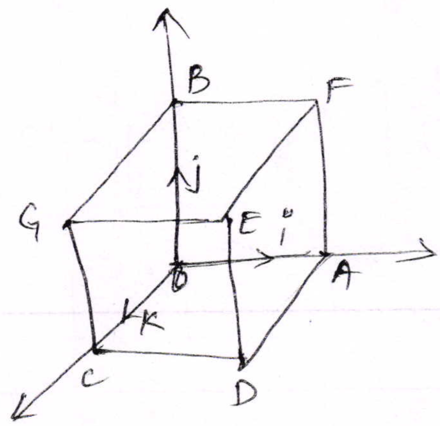
$$= \frac{32}{2} - 4 = 12$$

If $F = 4xz\,i - y^2 j + yz\,K$ evaluate

$\int F\cdot\hat{n}\, ds$ where S is the surface of the cube bounded by $x=0, y=0, z=0$ and $x=a, y=a, z=a$

Consider the given cube as shown in fig

(a) Consider the face OADC



On XZ plane $y=0$

$dy=0$

$\hat{n} = -j$

$ds = dx\,dz$

$F\cdot\hat{n} = y^2$

$$\int_{OADC} F\cdot\hat{n}\, ds = \int y^2\, dx\, dz = 0.$$

(b) for the face OAFB

On XY plane $z=0$

$dz=0$

$\hat{n} = -K$

$F\cdot\hat{n} = -yz$

$F\cdot\hat{n} = 0.$

$$\int_{OAFB} F\cdot\hat{n}\, ds = 0.$$

Ⓒ for the face OBGC.

on YZ plane, $x = 0$
$dx = 0$

$\hat{n} = -i$, $F \cdot \hat{n} = -4xz = 0$

$$\int_{OBGC} F \cdot \hat{n}\, ds = 0$$

ⓓ Consider the force BFEG.

$y = a$, $dy = 0$

$\hat{n} = j$

$ds = dx\,dz$

$F \cdot \hat{n} = -y^2$

$$\therefore \int_{R_4} F \cdot \hat{n}\, ds = -\int_0^a \int_0^a y^2\, dx\, dz$$

$$= -a^2 (x)_0^a (z)_0^a$$

$$= -a^4$$

ⓔ for the force GEDC

on XY plane $z = a$
$dz = 0$

$\hat{n} = K$, $ds = dx\,dy$

$F \cdot \hat{n} = yz = ay$

$$\therefore \int_{R_5} F \cdot \hat{n}\, ds = a \int_0^a \int_0^a y\, dx\, dy = a (x)_0^a \left(\frac{y^2}{2}\right)_0^a = \frac{a^4}{2}$$

(f) for the face EFAD

$$ds = dy\,dz \qquad x = a,\ dx = 0$$

$$\hat{n} = i \quad , \quad F\cdot\hat{n} = 4xz = 4az$$

$$\int_{R_6} F\cdot\hat{n}\,ds = 4a\int_0^a\int_0^a z\,dz\,dy$$

$$= \frac{4a}{2}\left(z^2\right)_0^a\left(y\right)_0^a$$

$$= 2a^4$$

$$\therefore \int_C F\cdot\hat{n}\,ds = 0+0+0+2a^4-a^4+\frac{a^4}{2}$$

$$= a^4+\frac{a^4}{2}$$

$$= \frac{3a^4}{2}$$

→ Evaluate $\int_S F\cdot\hat{n}\,ds$ where $F = 2xyi + yz^2j + xzK$. Find the Surface Integral over the parllelopiped $x=0,\ y=0,\ z=0,\ x=2,\ y=1,\ z=3$

(a) for face

VOLUME INTEGRALS

The process of finding integration along the given volume is Known as Volume Integration. It is denoted by

$\int_V F\,dV$.

Cartesian Form:

$$F = F_1 i + F_2 j + F_3 K.$$

$$dV = dx\,dy\,dz$$

$$\int_V F\,dV = i\iiint F_1\,dx\,dy\,dz + j\iiint F_2\,dx\,dy\,dz + K\iiint F_3\,dx\,dy\,dz.$$

→ Evaluate $\int_V F\,dV$ where $F = 2xzi - xj + y^2K$ such that V is the Region bounded by the surfaces $x=0,\ y=0,\ x=2,\ y=6,\ z=x^2,\ z=4$

$$F = 2xzi - xj + y^2K$$

$$F_1 = 2xz,\quad F_2 = -x,\quad F_3 = y^2$$

$$2i\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4} xz\,dx\,dy\,dz - j\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4} x\,dx\,dy\,dz + K\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4} y^2\,dx\,dy\,dz$$

$$= \frac{2i}{2}\int_0^2\int_0^6 x(z^2)_{x^2}^4\,dy\,dx - j\int_0^2\int_0^6 x(z)_{x^2}^4\,dy\,dx + K\int_0^2\int_0^6 y^2(z)_{x^2}^4\,dy\,dx$$

$$= i\int_0^2\int_0^6 x(16-x^4)\,dy\,dx - j\int_0^2\int_0^6 x(4-x^2) + K\int_0^2\int_0^6 y^2(4-x^2)\,dy\,dx$$

$+yz^2j+$

e the

$, y=1,$

$$i\int_0^2\int_0^6 (16x - x^5)\,dy\,dx - j\int_0^2\int_0^6 (4x - x^3)\,dy\,dx + k\int_0^2\int_0^6 (4y^2 - x^2y^2)\,dy\,dx.$$

$$i\int_0^2 16x(y)_0^6 - x^5(y)_0^6\,dx - j\int_0^2 4x(y)_0^6 - x^3(y)_0^6\,dx + k\int_0^2 \frac{4}{3}(y^3)_0^6 - \frac{x^2}{3}(y^3)_0^6\,dx$$

$$i\int_0^2 (96x - 6x^5)\,dx - j\int_0^2 (24x - 6x^3)\,dx + k\int_0^2 (288 - 72x^2)\,dx$$

$$= i\left(\frac{96}{2}(x^2)_0^2 - \frac{6}{6}(x^6)_0^2\right) - j\left(\frac{24}{2}(x^2)_0^2 - \frac{6}{4}(x^4)_0^2\right) + k\left(288(x)_0^2 - \frac{72}{3}(x^3)_0^2\right)$$

$$= i(192 - 64) - j(48 - 24) + k\left(288\times 2 - \frac{72}{3}\times 8\right)$$

$$= 128i + 24j + 384k$$

→ Evaluate $\int \nabla\cdot F\,dV$ where $F = (2x^2 - 3z)i - (2xy)j - 4xk$ where V is the closed Region bounded by $x=0,\ y=0,\ z=0,\ 2x+2y+z=4$

$$F = (2x^2 - 3z)i - 2xyj - 4xk$$

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\nabla\cdot f = \frac{\partial}{\partial x}(2x^2 - 3z) - \frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(4x)$$

$$= 4x - 0 - 2x + 0$$

$$\nabla\cdot f = 2x$$

$-x^2y^2 dy dx$

$$= 2\int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y} x\,dz\,dy\,dx$$

$2x+2y+z=4$

$z=4-2x-2y$

Put $z=0$

$2y=4-2x$

$y=2-x$

$x=2$

$)_0^6 - \frac{x^2}{3}(y^3)_0^6 dx$

$$= 2\int_0^2\int_0^{2-x} x\,(z)_0^{4-2x-2y}\,dy\,dx$$

$2x^2)dx$

$$= 2\int_0^2\int_0^{2-x} x(4-2x-2y)\,dy\,dx$$

$288(x)_0^2 - \frac{72}{3}(x^3)_0^2$

$\frac{2}{3}\times 8)$

$$= 2\int_0^2\int_0^{2-x}(4x-2x^2-2xy)\,dy\,dx$$

$-(2xy)j$

$$= 2\int_0^2\left[4x(y)_0^{2-x} - 2x^2(y)_0^{2-x} - \frac{2x}{2}(y^2)_0^{2-x}\right]dx$$

$2y+z=4$

$$= 2\int_0^2\left[4x(2-x)-2x^2(2-x)-x(2-x)^2\right]dx$$

$$= 2\int_0^2\left[8x-4x^2-4x^2+2x^3-x(4+x^2-4x)\right]dx$$

$$= 2\int_0^2\left[8x-8x^2+2x^3-4x-x^3+4x^2\right]dx$$

$$= 2\int_0^2\left[x^3-4x^2+4x\right]dx = 2\left[\frac{x^4}{4}-\frac{4x^3}{3}+\frac{4x^2}{2}\right]_0^2$$

$$= 2\left[\frac{16}{4}-\frac{4(8)}{3}+8\right] \Rightarrow 2\left[12-\frac{32}{3}\right] = 8/3$$

$$-\int_V \nabla\times f \, dV$$

$$\nabla\times f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2-3z & -2xy & -4x \end{vmatrix}$$

$$= i\left(-4\frac{\partial}{\partial y}(x) + 2\frac{\partial}{\partial z}(xy)\right) - j\left(-4(1) - \left[2\frac{\partial}{\partial z}(x^2) - 3\frac{\partial}{\partial z}(z)\right]\right)$$

$$+k\left[-2y\frac{\partial}{\partial x}(x) - \left[2\frac{\partial}{\partial y}(x^2) - 3\frac{\partial}{\partial y}(z)\right]\right]$$

$$= \cancel{i} \quad - j(-4+3) + k[-2y]$$

$$= \quad j - 2yk$$

$$F_2 = 1, \quad F_3 = -2y.$$

$$= j\int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y} 1 \, dz\, dy\, dx - 2k\int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y} y \, dz\, dy\, dx$$

$$= j\int_{x=0}^{2}\int_{y=0}^{2-x} \left[z\right]_0^{4-2x-2y} dy\, dx - 2k\int_{x=0}^{2}\int_{y=0}^{2-x} y(z)_0^{4-2x-2y} \, dy\, dx$$

$$= j\int_{x=0}^{2}\int_{y=0}^{2-x} (4-2x-2y)\, dy\, dx - 2k\int_{x=0}^{2}\int_{y=0}^{2-x} y(4-2x-2y)\, dy\, dx$$

$$= j\int_{x=0}^{2} 4(y)_0^{2-x} - 2x(y)_0^{2-x} - \frac{2}{2}(y^2)_0^{2-x} - 2k\int_0^2 \frac{4}{2}(y^2)_0^{2-x} - \frac{2x}{2}(y^2)_0^{2-x} - \frac{2}{2}(y^2)_0^{2-x}$$

$$j\int_0^2 \left(4(2-x)-2x(2-x)-(2-x)^2\right)dx$$

$$-2K\int_0^2 2(2-x)^2 - x(2-x)^2 - (2-x)^2\, dx.$$

→ If S is a closed region in xy plane bounded by a simple closed curve C and if M, N are continuous functions of x and y and having continuous derivatives in R then

$$\oint M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

where C is traversed in the positive direction

→ Evaluate by $\int_C (2xy - x^2)dx + (x^2+y^2)dy$

C is the closed curve bounded by $y = x^2$; $x^2 = y$

$$\int_C (2xy - x^2)dx + (x^2+y^2)dy.$$

$$= \int_0^1 \int_{y=x^2}^{y=\sqrt{x}} [2x - 2x]\, dx$$

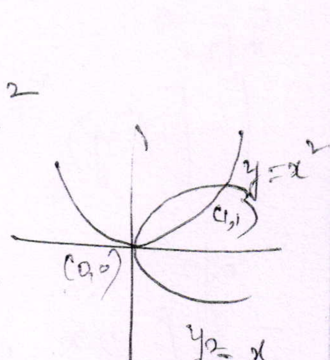
$$= 0$$

$M = 2xy - x^2$

$\frac{\partial M}{\partial y} = 2x$

$N = x^2 + y^2$

$\frac{\partial N}{\partial x} = 2x$



$\frac{\partial}{\partial z}(x^2) - \frac{\partial}{\partial z}(\ )$

x

$y)\,dy\,dx$

$y^2)_0^{2-x} - \frac{2}{2}(y^2)_0^{2-x}$

→ Apply this theorem

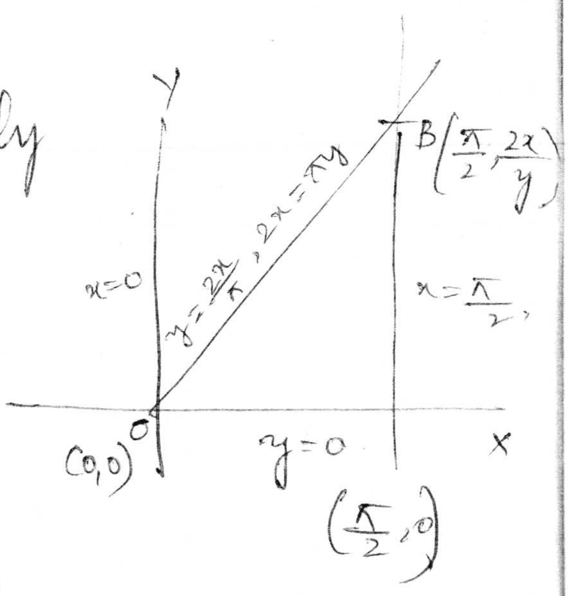
$\int_C (y - \sin x)\,dx + \cos x\,dy$ where C is the triangle enclosed by the lines

$y = 0$, $x = \frac{\pi}{2}$, $2x = \pi y$.

$M = y - \sin x$, $N = \cos x$

$\frac{\partial M}{\partial y} = 1$, $\frac{\partial N}{\partial x} = -\sin x$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$



$$\iint (-\sin x - 1)\,dx\,dy$$

$$= -\int_0^{\pi/2}\int_0^{\frac{2x}{\pi}} (1 + \sin x)\,dx\,dy$$

$$= -\int_0^{\pi/2} \left[(y)_0^{\frac{2x}{\pi}} + \sin x\,(y)_0^{\frac{2x}{\pi}} \right] dx$$

$$= -\int_0^{\pi/2} \left[\frac{2x}{\pi} + \frac{2x \sin x}{\pi} \right] dx$$

$= -\frac{2}{\pi}$

$= -\frac{2a}{\pi}$

$= \frac{-2}{\pi}$

$$= -\frac{2}{\pi}\left[\int_0^{\pi/2} x\,dx + \int_0^{\pi/2} x \sin x\,dx\right]$$

$$= -\frac{2}{\pi}\left[\left(\frac{x^2}{2}\right)_0^{\pi/2} + x(-\cos x) - \int(-\cos x)\,dx\right]$$

$$= \frac{-2}{\pi}\left[\frac{\pi^2}{8} - x\cos x + \sin x\right]$$

$B\left(\frac{\pi}{2}, \frac{2x}{y}\right)$

$x = \frac{\pi}{2}$,

x

$$\int_{0}^{\pi/2}\int_{-1}^{1} x^2 y^2\,dx\,dy$$

$$dy \cdot \int_{-1}^{1} x^2\,dx.$$

$$\left.\frac{3}{3}\right]_0^{\pi/2} \cdot \left[\frac{x^3}{3}\right]_{-1}^{1}$$

$$\left.\frac{)^3}{3}\right] \cdot \left[\frac{1}{3}+\frac{1}{3}\right]$$

$$\frac{\pi^3}{24}\left(\frac{2}{3}\right) = \frac{\pi^3}{36}.$$

$$\int_0^5\int_0^{x^2} x\,(x^2+y^2)\,dx\,dy.$$

$$\int_0^2 (x^3+xy^2)\,dx\,dy.$$

$$\cancel{\int_0^{x^2} xy^2} = \int_0^5 \left(x^3 (y)_0^{x^2} + x\left(\frac{y^3}{3}\right)_0^{x^2} \right) dx.$$

$$= \int^5 \left(x^5 + x\cdot\frac{x^6}{3} \right) dx.$$

=

=

=

$\longrightarrow$

=

a

$= \int$

$$= \int_0^5 \left(x^5 + \frac{x^7}{3}\right) dx$$

$$= \int_0^5 x^5\, dx + \frac{1}{3}\int_0^5 x^7\, dx$$

$$= \left(\frac{x^6}{6}\right)_0^5 + \frac{1}{3}\left(\frac{x^8}{8}\right)_0^5$$

$$= \cancel{\frac{15625}{6} + \frac{1}{3}\left(\frac{78125}{7}\right)}$$

$$= \frac{5^6}{6} + \frac{1}{3}\left(\frac{5^8}{8}\right)$$

$$= \frac{5^6}{3}\left(\frac{1}{2} + \frac{25}{8}\right)$$

$$= \frac{5^6}{3}\left(\frac{4+25}{8}\right) \Rightarrow \frac{5^6}{3}\left(\frac{29}{8}\right)$$

$$= 5^6\left(\frac{29}{24}\right)$$

$$\rightarrow \int_0^a \int_0^{\sqrt{a^2-x^2}} \left(\sqrt{a^2-x^2-y^2}\right) dy\, dx.$$

$$\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

$$= \int_0^a \left[\int_0^{\sqrt{a^2-x^2}} \sqrt{(a^2-x^2) - y^2}\, dy\right] dx.$$

$$= \int_0^a \left[\left[\frac{y}{2}\sqrt{(a^2-x^2)-y^2} + \frac{(a^2-x^2)}{2}\sin^{-1}\left(\frac{y}{\sqrt{a^2-x^2}}\right)\right]\right]^{\sqrt{a^2-x^2}} dx$$

$$\left[\frac{\sqrt{a^2-x^2}}{2}\sqrt{(a^2-x^2)-(a^2-x^2)}+\frac{(a^2-x^2)}{2}\sin^{-1}\left(\frac{\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}}\right)\right]dx.$$

$$\int\left(\frac{(a^2-x^2)}{2}\cdot\frac{\pi}{2}\right)dx.$$

$$\int(a^2-x^2)\,dx,$$

$$\left[\left(a^2(x)_0^a-\left(\frac{x^3}{3}\right)_0^a\right]\cancel{dx}$$

$$\frac{\pi}{4}\left[a^2(a)-\frac{a^3}{3}\right]\Rightarrow\frac{\pi a^3}{4_2}\left(\frac{\cancel{2}}{3}\right)$$

$$=\frac{\pi a^3}{6}.$$

$$\int^{x}(x^2+y^2)\,dx\,dy.$$

$$\cancel{R}\ y=0$$

the +ve Quadrant for which $x+y\leq 1$

$$\int\left[x^2(y)_0^{1-x}+\frac{(y^3)_0^{1-x}}{3}\right]dx.$$

$$\int_0^1\left[x^2(1-x)+\frac{(1-x)^3}{3}\right]dx,$$

$=$

$\int_0^{\infty} e$

$\int_0^{\infty} e$

$\frac{1}{2}$

$$= \int_0^1 \left[x^2 - x^3 + \frac{1 \cancel{+} x^3 - 3x + 3x^2}{3}\right]$$

$$\int_0^1 \left[x^2 - x^3 + \frac{1}{3} - \frac{x^3}{3} - x + x^2\right]$$

$$= \left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1 + \frac{1}{3}[x]_0^1 - \frac{1}{3}\left[\frac{x^4}{4}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x^3}{3}\right]_0^1$$

$$\rightarrow \int_0^\infty \int_0^{\pi/2} e^{-r^2} \cdot r \, d\theta \, dr$$

$$\int_0^\infty e^{-r^2} \cdot r \, dr \cdot \int_0^{\pi/2} d\theta$$

Put $r^2 = t \Rightarrow 2r\,dr = dt$

$$= \int_0^\infty e^{-t} \cdot \frac{dt}{2} \cdot \int_0^{\pi/2} d\theta$$

$$= \frac{-1}{2}\left(e^{-t}\right)_0^\infty \cdot (\theta)_0^{\pi/2}$$

$$= \frac{-1}{2}\left(e^{-\infty} - e^{0}\right) \cdot \left(\frac{\pi}{2}\right)$$

$$= -\frac{\pi}{4}(0-1) = \frac{\pi}{4}$$

Working rule to find the polar limits of integration

Consider the integral $\iint_R f(r,\theta)\,dr\,d\theta$ over the Region R where the limits of integration are not given

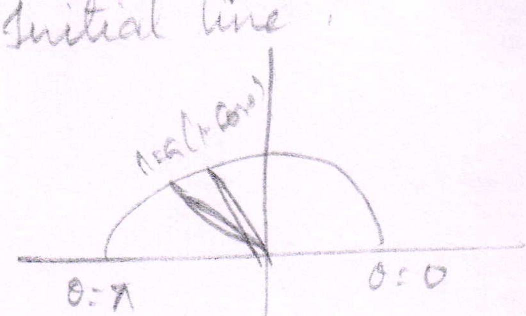
Step 1: Sketch the Region of integration R and label the boundaries of the curve.

Step 2: Imagine a Radius Vector OA through the Region Mark the Values of R in terms of θ

Step 3: Find the smallest & largest Values of θ which includes the Region R this gives the limits for θ.

Therefore the limits of θ integration Varies from θ_1 to $\theta = \theta_2$

→ Evaluate $\iint_R r \sin\theta\,dr\,d\theta$ where the Region is the cardioid $r = a(1-\cos\theta)$ above the Initial line.



$$\int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=a(1-\cos\theta)} r\sin\theta\,dr\,d\theta$$

$$\int^{\pi}\sin\theta\cdot d\theta\cdot\int^{r=a(1-\cos\theta)} r\,dr$$

$$= \left(-\cos\theta\right)_0^{\pi} \cdot \left(\frac{r^2}{2}\right)_0^{a(1-\cos\theta)}$$

$$= \left(-\cos\pi + \cos 0\right)\left(\frac{a^2(1-\cos\theta)^2}{2}\right)$$

$$= \left(-(-1)+1\right)\left(\frac{a^2(1-\cos\theta)^2}{2}\right)$$

$$= a^2(1-\cos\theta)^2$$

$$\frac{1}{2}\int_0^{\pi}\left(a^2\right)_0^{a(1-\cos\theta)} \sin\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} a^2(1-\cos\theta)^2 \sin\theta \, d\theta$$

$1-\cos\theta = t$

~~$a + \sin\theta \, d\theta = dt$~~

$1-\cos\theta = t$

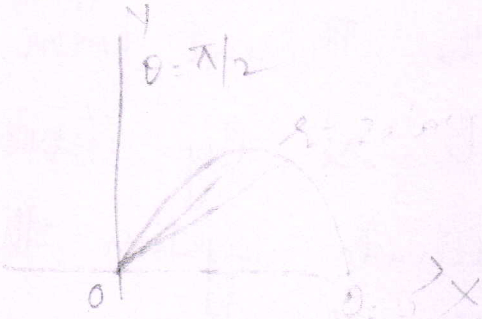
$+\sin\theta \, d\theta = dt$

$$\frac{a^2}{2\times 3}\left[(1-\cos\theta)^3\right]_0^{\pi}$$

$$= \frac{a^2}{6}\left[(1-\cos\pi)^3 - (1-\cos 0)^3\right]$$

$$= \frac{a^2}{6}\left[8\right] = \frac{4a^3}{3}$$

Evaluate $\iint_R r^2 \sin\theta \, dr \, d\theta$ where R is the Region semicircle $r = 2a\cos\theta$ above the initial line



$$\int_0^{\pi/2} \int_0^{2a\cos\theta} r^2 \sin\theta \, dr \, d\theta$$

$$\frac{1}{3} \int_0^{\pi/2} \left[r^3 \right]_0^{2a\cos\theta} \cdot \sin\theta \cdot d\theta$$

$$\frac{1}{3} \int_0^{\pi/2} 8a^3 \cos^3\theta \cdot \sin\theta \cdot d\theta$$

$\cos\theta = t$

$-\sin\theta \cdot d\theta = dt$

$\cos 0 = t \Rightarrow t = 1$

$\cos \pi/2 \Rightarrow t = 0$

$$-\frac{8a^3}{3} \int_1^0 t^3 \cdot dt$$

$$\frac{8a^3}{3} \int_0^1 t^3 \, dt = \frac{8a^3}{3} \left(\frac{t^4}{4} \right)_0^1$$

$$= \frac{8a^3}{3} \cdot \frac{1}{4} = \frac{2a^3}{3}$$

Change of Variables in Double Integrals

Some types evaluation of double Integral with the Present form may not be simple to evaluate, So By choice of an appropriate co-ordinate system the given Integral can be transformed into a simpler Integral having new Variables.

Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dx\, dy$ into polar Co-ordinates

Also find $\int_0^{\infty} e^{-x^2} dx$.

$$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dx\, dy. \Rightarrow$$

Put $x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

θ Varies from 0 to $\pi/2$

r Varies from 0 to ∞

$$\int_0^{\infty}\int_0^{\infty} e^{-x^2} \cdot e^{-y^2} dx\, dy$$

$$\int_0^{\infty} e^{-y^2} dy \cdot \int_0^{\infty} e^{-x^2} dx.$$

$$\int_0^{\pi/2}\int_0^{\infty} e^{-r^2} \cdot r\, dr\, d\theta.$$

$$= \int_0^{\pi/2} d\theta \cdot \int_0^{\infty} e^{-r^2} \cdot r\,dr \qquad r^2 = t$$

$$2r\,dr = dt$$

$$= [\theta]_0^{\pi/2} \cdot \frac{1}{2}\int_0^{\infty} e^{-t} \cdot dt$$

$$= \frac{\pi}{4}\left[-e^{-t}\right]_0^{\infty}$$

$$= \frac{\pi}{4}\left[-e^{-\infty} + e^{-0}\right]$$

$$= \frac{\pi}{4}.$$

$$I = \left[\int_0^{\infty} e^{-x^2} dx\right]\left[\int_0^{\infty} e^{-y^2} dy\right]$$

$$I = \left[\int_0^{\infty} e^{-x^2} dx\right]^2 = \frac{\pi}{4}.$$

$$\therefore \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

→ Evaluate $\displaystyle\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\, dx\,dy.$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2 \to (1)$

$dx\,dy = r\,dr\,d\theta.$

$x = 0,\ x = a$

$y = 0,\ y = \sqrt{a^2-x^2}$

$y^2 = a^2 - x^2$

$x^2 + y^2 = a^2 \to (2)$

$r^2 = a^2 \to r = a$

$\cos^{-1}\left(\frac{a}{r}\right)$

$$\int_{\theta=0}^{\pi/2}\int_0^{a} r\sin\theta \cdot r \cdot r\,dr\,d\theta$$

$$\int_0^a r^3 \sin\theta \, dr \, d\theta$$

$$\sin\theta \, d\theta \cdot \int_0^a r^3 \, dr$$

$$\left(\cos\theta\right)_0^{\pi/2} \cdot \left(\frac{r^4}{4}\right)_0^a$$

$$\left(\cos\frac{\pi}{2} + \cos 0\right)\left(\frac{a^4}{4}\right)$$

$$\frac{a^4}{4}$$

Evaluate $\displaystyle\int_{y=0}^{a}\int_{x=0}^{\sqrt{a^2-y^2}} (x^2+y^2)\, dx\, dy.$

$x = r\cos\theta$
$y = r\sin\theta$
$x^2+y^2 = r^2$
$dx\,dy = r\,dr\,d\theta$

$x = \sqrt{a^2-y^2}$
$x^2+y^2 = a^2$
$r = a$

$$\int_0^{\pi/2}\int_{r=0}^{a} (r^2)\cdot r\,dr\,d\theta$$

$$\left[\theta\right]_0^{\pi/2} \left[\frac{r^4}{4}\right]_0^a$$

$$\frac{\pi}{2} \cdot \frac{a^4}{4} = \frac{\pi a^4}{8}$$

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)\,dx\,dy$$

$y = \sqrt{2x-x^2}$

$y^2+x^2-2x=0$

$x^2+y^2=r^2$

$r^2-2x=0$

$r^2=2x$

$r=\sqrt{2}\sqrt{x}$

$r^2=2r\cos\theta$

$r=2\cos\theta$

$$\int_0^{\pi/2}\int_0^{2\cos\theta} r^2 \cdot r\,dr\,d\theta$$

$$[\theta]_0^{\pi/2}\left[\frac{r^4}{4}\right]_0^{2\cos\theta}$$

$$= \frac{\pi}{8}\cdot 16\cos^4\theta$$

$$= 2\pi\cos^4\theta$$

$$= \frac{1}{4}\int_0^{\pi/2}\left(r^4\right)_0^{2\cos\theta} = \frac{1}{4}\int_0^{\pi/2} 16\cos^4\theta$$

$$= 4\int_0^{\pi/2}\cos^4\theta$$

$$= 4\int_0^{\pi/2}\cos^2\theta\,(1-\sin^2\theta)$$

$$= 4\left[\int_0^{\pi/2}\cos^2\theta - \frac{1}{4}\int_0^{\pi/2} 4\cos^2\theta\sin^2\theta\right]$$

$$= 4\left[\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right) - \frac{1}{4}\int_0^{\pi/2}\left(\frac{1-\cos 4\theta}{2}\right)\right]$$

$$= 4\left(\frac{1}{2}\right)_0^{\pi/2} + \left(\frac{\sin 2\theta}{4}\right)_0^{\pi/2} - \left[\left(\frac{1}{2}\right)_0^{\pi/2} - \left(\frac{\sin 4\theta}{8}\right)_0^{\pi/2}\right]$$

$$\int_0^{4a}\int_{\frac{y^2}{4a}}^{y} \frac{x^2-y^2}{x^2+y^2}\,dx\,dy = 8a^2\left(\cancel{4\pi/2}\right)$$

$$8a^2\left(\frac{\pi}{2}-\frac{5}{3}\right)$$

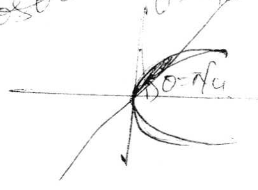
$$\int_0^{\frac{4a\cos\theta}{\sin^2\theta}} \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r^2\cos^2\theta + r^2\sin^2\theta}\cdot r\cdot dr\cdot d\theta$$

$x=\frac{y^2}{4a}$

$8a\cdot r\cos\theta = r$

$\theta:\pi/2$

$\theta=\pi/4$



$y^2=4ax$

$r^2\sin^2\theta = 4a\,r\cos\theta$

~~$\tan\theta\cdot\sin\theta$~~

$\frac{4a\cos\theta}{\sin^2\theta}$

$$\int_0^{\frac{4a\cos\theta}{\sin^2\theta}} \cos 2\theta\cdot r\cdot dr\cdot d\theta\cdot$$

$$\cos 2\theta\left[r^2\right]_0^{\frac{4a\cos\theta}{\sin^2\theta}}$$

$$\int_0^{\pi/2}\cos 2\theta\cdot\frac{\overset{8}{\cancel{16}}a^2\cos^2\theta}{\sin^4\theta}$$

$$a^2\int^{\pi/2}\frac{\cos^2\theta\cdot\cos 2\theta}{\sin^4\theta}$$

$$a^2\int_0^{\pi/2}\frac{\left(\frac{1+\cos 2\theta}{2}\right)\cos 2\theta}{\sin^2\theta\,(1-\cos^2\theta)} \Rightarrow 8a^2\int_0^{\pi/2}\frac{\frac{\cos 2\theta}{2}+\frac{1+\cos 4\theta}{4}}{\left(\frac{1-\cos 2\theta}{2}\right)-\frac{1}{4}\left(\frac{1-\cos 4\theta}{2}\right)}$$

$$= 16a^2\int_0^{\pi/2}\frac{2\cos 2\theta+1+\cos 4\theta}{4-4\cos 2\theta-1+\cos 4\theta}$$

$$= 16a^2\int_0^{\pi/2}\frac{\cos 4\theta+2\cos 2\theta+1}{\cos 4\theta-4\cos 2\theta+3}$$

→ ... that ... i.e., $\int^{b}$ ... $x=a$ ... Int

→ Eva ... $\int_0^{4a}$

→ The Change of Order of Integration implies that the change of limits of Integration

i.e. $\int_{x=a}^{b}\int_{y=f_1(x)}^{y=f_2(x)} f(x,y)\,dy\,dx$ by changing the Order of Integration converts into $\int_{y=a}^{b}\int_{x=f_1(y)}^{x=f_2(y)} f(x,y)\,dx\,dy$.

→ Evaluate by Changing the Order of Integration.

$$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$$

$$\int_{y=0}^{y=4a}\int_{x=\frac{y^2}{4a}}^{x=2\sqrt{ay}} dx\,dy.$$

$$\int_0^{4a} (x)_{\frac{y^2}{4a}}^{2\sqrt{ay}}\,dy = \int_0^{4a}\left[2\sqrt{ay} - \frac{y^2}{4a}\right]dy.$$

$$= 2\sqrt{a}\int_0^{4a}\sqrt{y}\,dy - \frac{1}{4a}\int_0^{4a} y^2\,dy$$

$$= 2\sqrt{a}\cdot\frac{2}{3}\left[y^{3/2}\right]_0^{4a} - \frac{1}{12a}\left(y^3\right)_0^{4a}$$

$$= \frac{4}{3}\sqrt{a}\left((4a)^{3/2}\right) - \frac{1}{12a}\cdot 64a^3 \qquad \frac{32a^2}{3} - \frac{16a^2}{3}$$

$$= \frac{32}{3}a\sqrt{a^2} - \frac{16a^2}{3} \Rightarrow \cancel{\frac{16a}{3}\left[2\sqrt{a} - a\right]} = \frac{16a^2}{3}.$$

hange the Order of Integration

ate $\int_{x=0}^{a}\int_{y=x/a}^{\sqrt{x/a}} (x^2+y^2)\,dx\,dy$.

$\int \int_{y/a}^{\sqrt{y/a}} (x^2+y^2)\,dy\,dx$.

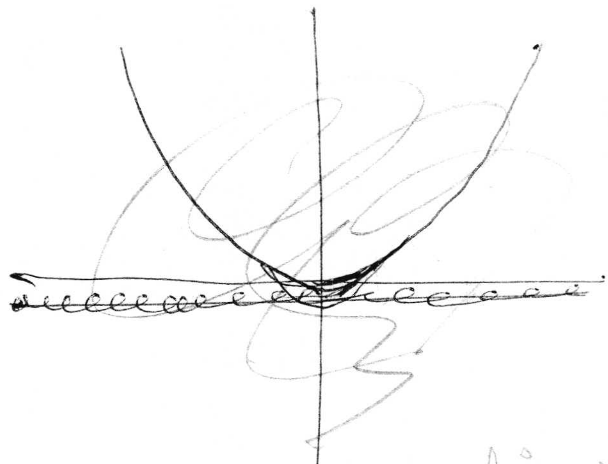
Change of Order of Integration

$\int_{x^2/2a}^{2a-x} xy^2\,dy\,dx$

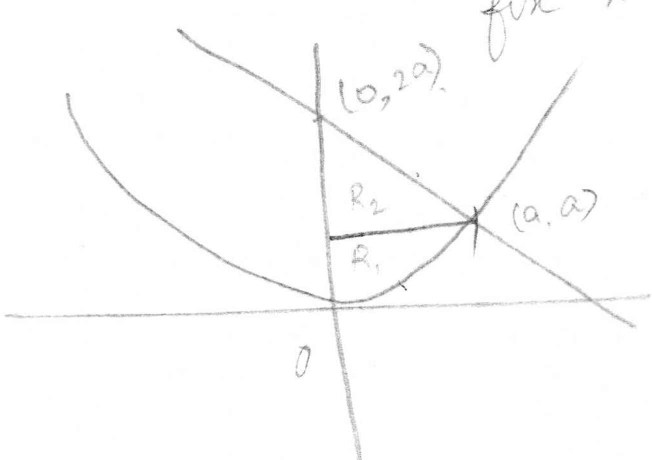
$=0$ to $x=a$

$\frac{x^2}{a}$ to $y=2a-x$.

$=ay$ $\quad x+y=2a$.



fix x



a

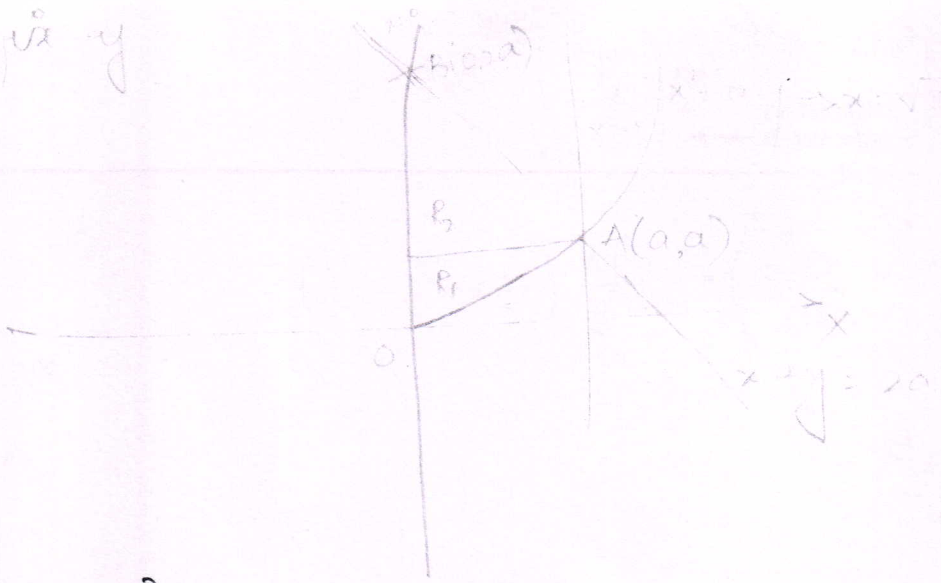
0

$\int_0^a y^2$

$= \int_0^a$

$= \frac{a}{2}$

$\frac{a}{2}\Big[$



$$x^2 = ay \;, \quad y = 2a - x$$

$$x^2 = a(2a - x)$$

$$x^2 = 2a^2 - ax$$

$$x^2 + ax - 2a^2 = 0$$

$$\int_0^a \int_{y=\frac{x^2}{a}}^{2a-x} x y^2 \, dy \, dx = \int_0^a \int_0^{\sqrt{ay}} x y^2 \, dx \, dy + \int_0^{2a} \int_0^{2a-y} x y^2 \, dx \, dy$$

$$\int_0^a y^2 \left(\frac{x^2}{2}\right)_0^{\sqrt{ay}} dy + \int_0^{2a} y^2 \left[\frac{x^2}{2}\right]_0^{a-y} dy$$

$$\int_0^a y^2 \left(\frac{ay}{2}\right) \cdot dy + \int_0^{2a} y \frac{(2a-y)^2}{2}$$

$$\frac{a}{2} \int_0^a y^3 \cdot dy + \int_0^{2a} y^2 \frac{(4a^2 + y^2 - 4ay)}{2}$$

$$\frac{a}{2}\left[\frac{y^4}{4}\right]_0^a + \int_0^{2a} \left(\frac{4a^2}{2} \cdot y^2 + \frac{y^4}{2} - \frac{4ay^3}{2}\right) dy$$

$$\left(\frac{a^4}{4}\right) + 2a^2\left(\frac{y^3}{3}\right)_a^{2a} + \left(\frac{y^5}{10}\right)_a^{2a} - 4a\left(\frac{y^4}{8}\right)_a^{2a}$$

$$+ 2a^2\left[\frac{8a^3 - a^3}{3}\right] + \left(\frac{32a^5}{10} - \frac{a^5}{10}\right) - \frac{4a}{8}\left(16a^4 - a^4\right)$$

$$+ \frac{14a^5}{3} + \frac{31a^5}{10} - \frac{60a^5}{8}$$

$$\frac{47a^5}{120}$$

ey Changing the Order of Integration

$$\int_{0}^{\infty} \frac{e^{-5}}{y}\,dy\,dx.$$

$$\int_{y}^{\infty} \frac{e^{-5}}{y}\,dx\,dy$$

$\int_{x=0}^{a}$

$\int_{x=0}^{a}$

$\int_{x=0}^{a}$

$\int_{x=0}^{a}$

$\int_{x=0}^{a}$

$= \frac{1}{2}$

x

$= \frac{1}{2}$

x

Triple Integration

$$\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}}\int_{z=0}^{\sqrt{a^2-x^2-y^2}} x y^2 z \, dz\, dy\, dx$$

$$\int_{x=0}^{a}\left[\int_{y=0}^{\sqrt{a^2-x^2}} \frac{x y^2}{2}\left(z^2\right)_0^{\sqrt{a^2-x^2-y^2}} dy\right] dx$$

$$\int_{x=0}^{a}\left[\int_{y=0}^{\sqrt{a^2-x^2}} \frac{x y^2}{2}\left(\left(\sqrt{a^2-x^2-y^2}\right)^2\right) dy\right] dx$$

$$\int_{x=0}^{a}\left[\frac{1}{2}\int_{y=0}^{\sqrt{a^2-x^2}} x y^2\left(a^2-x^2-y^2\right) dy\right] dx$$

$$\int_{x=0}^{a}\left[\frac{1}{2}\int_{y=0}^{\sqrt{a^2-x^2}}\left(a^2 x y^2 - x^3 y^2 - x y^4\right) dy\right] dx.$$

$$\frac{1}{2}\int_{x=0}^{a}\left[\frac{a^2 x y^3}{3} - \frac{x^3 y^3}{3} - \frac{x y^5}{5}\right]_0^{\sqrt{a^2-x^2}} dx.$$

$$\frac{1}{2}\int_{x=0}^{a}$$

$$\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}}\int_{z=0}^{\sqrt{a^2-x^2-y^2}} xy^2z \, dz\, dy\, dx$$

$$\int_{x=0}^{a}\left[\int_{y=0}^{\sqrt{a^2-x^2}} \frac{xy^2}{2}\left(z^2\right)_0^{\sqrt{a^2-x^2-y^2}} dy\right] dx$$

$$\int_{x=0}^{a}\left[\int_{y=0}^{\sqrt{a^2-x^2}} \frac{xy^2}{2}\left[\sqrt{a^2-x^2-y^2}\right]^2 dy\right] dx.$$

$$\frac{1}{2}\int_{x=0}^{a}\left[x\int_{y=0}^{\sqrt{a^2-x^2}}\left(y^2(a^2-x^2)-y^4\right)dy\right]dx.$$

$$\frac{1}{2}\int_{x=0}^{a} x\left[\frac{(a^2-x^2)y^3}{3}-\frac{y^5}{5}\right]_0^{\sqrt{a^2-x^2}} dx.$$

$$\frac{1}{2}\int_{x=0}^{a} x\left[\frac{(a^2-x^2)(a^2-x^2)^{3/2}}{3}-\frac{(a^2-x^2)^{5/2}}{5}-0\right]dx.$$

$\iiint dz\,dy\,dz$ where V is the finite region of space formed by the planes $x=0, y=0, z=0, 2x+3y+4z=12$.

$$4z = 12-2x-3y$$

$$z = \frac{12-2x-3y}{4}$$

Put $z=0$

$$3y = 12-2x$$

$$y = \frac{12-2x}{3}$$

Put $y=0$

$$2x = 12$$

$$x = 6$$

$$\int_0^6 \int_0^{\frac{12-2x}{3}} \int_0^{\frac{12-2x-3y}{4}} dz\,dy\,dx.$$

$$= \int_0^6 \int_0^{\frac{12-2x}{3}} [z]_0^{\frac{12-2x-3y}{4}} dy\,dz.$$

$$= \int_0^6 \int_0^{\frac{12-2x}{3}} \left[\frac{12-2x-3y}{4}\right] dy\,dz.$$

$$= \int_0^6 \left[3y - \frac{x}{2}\cdot y - \frac{3}{4}\cdot\frac{y^2}{2}\right]_0^{\frac{12-2x}{3}}$$

$$\Rightarrow \frac{1}{4}\int_0^6 \left[\left(\frac{12-2x}{3}\right)(12-2x) - \frac{3}{2}\left(\frac{12-2x}{3}\right)^2\right] dx.$$

$$= \frac{1}{4\times 3}\int_0^6 \left[(12-2x)^2 - \frac{(12-2x)^2}{2}\right] dx.$$

$$= \frac{1}{12\times 2}\int_0^6 (12-2x)^2\, dx.$$

$$= \frac{1}{24}\left[144(x)_0^6 + \frac{4}{3}(x^3)_0^6 - \frac{48}{2}(x^2)_0^6\right]$$

$$= \frac{1}{24}\left[144(6) + \frac{4}{\cancel{3}}(\cancel{216}^{72}) - \frac{48}{\cancel{2}}(\cancel{36}^{18})\right]$$

$$= \frac{1}{24}\left[\cancel{864} + 288 - \cancel{864}\right]$$

$$= \frac{288}{24} = 12.$$

Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y} \frac{dz\,dy\,dx}{(x+y+z+1)^3}$

$$\int_0^1\int_0^{1-x}\int_0^{1-x-y} (x+y+z+1)^{-3}\,dz\,dy\,dx$$

$$\int_0^1\int_0^{1-x}\left[\frac{(x+y+z+1)^{-2}}{-2}\right]_0^{1-x-y} dy\,dx$$

$$\int_0^1\int_0^{1-x} (\cancel{x}+\cancel{y}+1-\cancel{x}-\cancel{y}+1)^{-2}\,dy\,dz$$

$$\frac{1}{8}\int_0^1\int_0^{1-x} dy\,dx$$

$$\frac{-1}{8}\int_0^1 [y]_0^{1-x}\,dx$$

$$= \frac{-1}{8}\int_0^1 (1-x)\,dx$$

$$= \frac{-1}{8}\left[x-\frac{x^2}{2}\right]_0^1$$

$$= \frac{-1}{8}\left[1-\frac{1}{2}\right] = \frac{-1}{16}$$

plications of Multiple Integrals:

rea Enclosed by a Plain Curve.

nsider the Area enclosed by the Curves $y = f(x)$ and $= g(x)$ and the Straight lines $x = a$, $x = b$.

e Area of the Region R is denoted by

$$A = \iint_R f(x, y)\, dx\, dy$$

$$= \iint_R f(x, y)\, dy\, dx$$

d the Area enclosed by the parabolas $y^2 = x$ and $x^2 = y$

$$y^2 = x, \quad x^2 = y \longrightarrow ②$$

$$y^4 = x^2 \longrightarrow ①$$

sub ① in ②

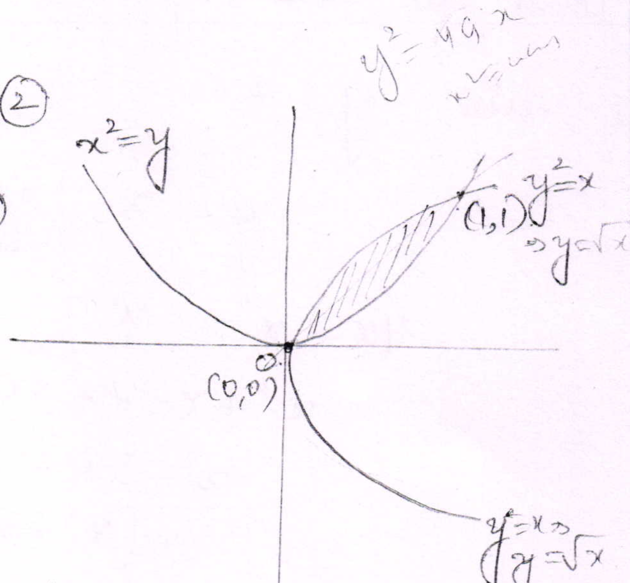
$$y^4 = y$$

$$\Rightarrow y(y^3 - 1) = 0$$

$$\Rightarrow y = 0, 1$$

at $y = 0 \Rightarrow x = 0$

$y = 1 \Rightarrow x = 1$ $\therefore (0,0), (1,1)$



$$A = \int_{x=0}^{x=1} \int_{y=x^2}^{y=\sqrt{x}} 1\, dy\, dx.$$

Note

$\rightarrow$

x^2

$\rightarrow$

th

$$= \int_0^1 (y)_{x^2}^{\sqrt{x}}\, dx$$

$$= \int_0^1 (\sqrt{x} - x^2)\, dx$$

$$= \frac{2}{3}\left(x^{3/2}\right)_0^1 - \frac{1}{3}\left(x^3\right)_0^1$$

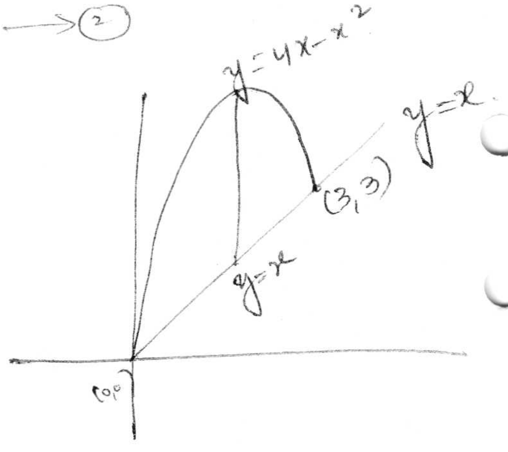
$$= \frac{2}{3} - 1/3 = 1/3.$$

Note:

→ The Area formed by the Parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $A = \left|\frac{16\,ab}{3}\right|$

→ Find the area of the parabola $y = 4x - x^2$ and the line $y = x$.

$$y = x, \quad y = 4x - x^2 \longrightarrow (2)$$



$$4x - x^2 = x$$

$$x^2 + x - 4x = 0$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0,\ x = 3$$

At $x = 0 \Rightarrow y = 0$

$x = 3 \Rightarrow y = 3$

(0,0) , (3,3).

$$A = \int_0^3 \int_x^{4x-x^2} dy\, dx.$$

$$\int_0^3 \left[y\right]_x^{4x-x^2} dx$$

$$\int_0^3 (4x-x^2-x)\,dx$$

$$\int_0^3 (3x-x^2)\,dx$$

$$\frac{3}{2}\left[x^2\right]_0^3 - \frac{1}{3}\left[x^3\right]_0^3$$

$$\frac{3}{2}[9] - \frac{1}{3}[27]$$

$$= 9\left[\frac{3}{2}-1\right] = \frac{9}{2}.$$

ind the double Integration and thearea enclosed

$(2-x)$ and $y = 2-x$

$$y^2 = 2(2-x), \quad y = 2-x$$

$$y^2 = 2y$$

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

$$y=0, \; y=2$$

At $y=0 \Rightarrow x=2.$

$y=2 \Rightarrow x=0$ $\qquad (0,2), (2,0)$

Volume

$\iint dV =$

Find

the

$$\int_{x=0}^{a} \int_{y=}$$

$$\int_0^a \int_0^{b}$$

$$\int_0^a \int_0^{b}$$

$$c\int_0^a \int_0^{b}$$

$$c\int_0^a$$

$$c\int_0^a$$

olume as triple Integral
The total Volume of the solid is $\int dv$

$$dv = \iiint_V dx\,dy\,dz$$

Find the Volume of the tetrahedron bounded by the planes $x=0$, $y=0$, and $z=0$, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$$\int_{x=0}^{a}\int_{y=0}^{b(1-\frac{x}{a})}\int_{z=0}^{c(1-\frac{x}{a}-\frac{y}{b})} dz\,dy\,dx$$

$$\frac{z}{c} = 1-\frac{x}{a}-\frac{y}{b}$$

$$z = c\left(1-\frac{x}{a}-\frac{y}{b}\right)$$

$$\int_0^a\int_0^{b(1-\frac{x}{a})} [z]_0^{c(1-\frac{x}{a}-\frac{y}{b})}\,dy\,dx.$$

Put $z=0$

$$y = b\left(1-\frac{x}{a}\right).$$

Put $y=0$

$$\frac{x}{a}=1$$

$$x=a.$$

$$\int_0^a\int_0^{b(1-\frac{x}{a})} c\left(1-\frac{x}{a}-\frac{y}{b}\right)dy\,dx.$$

$$c\int_0^a\int_0^{b(1-\frac{x}{a})}\left(1-\frac{x}{a}-\frac{y}{b}\right)dy\,dx$$

$$c\int_0^a\left[y-\frac{xy}{a}-\frac{y^2}{2b}\right]_0^{b(1-\frac{x}{a})}dx.$$

$$= c\int_0^a\left[b\left(1-\frac{x}{a}\right)-\frac{xb}{a}\left(1-\frac{x}{a}\right)-\frac{1}{2b}\cdot b^2\left(1-\frac{x}{a}\right)^2\right]dx.$$

$$\int_5^9 \sqrt[4]{(9-x)(x-5)}\,dx = \frac{2\left(\Gamma\frac{1}{4}\right)^2}{3\sqrt{\pi}}$$

$$\int_a^b (x-a)^m (b-x)^n\,dx \quad \text{then} \quad x-a = (b-a)t$$

$$\therefore \quad x-5 = (9-5).t$$

$$x-5 = 4t \Rightarrow x = 5+4t \qquad U.L \Rightarrow t = 1$$

$$dx = 4dt \qquad L.L \Rightarrow t = 0$$

$$(4t)^{1/4}(9-4t-5)^{1/4}.\ 4dt$$

$$\int_0^1 4^{1/4}.t^{1/4}(4-4t)^{1/4}.dt$$

$$= \int_0^1 4^{1/4}.4^{1/4}.t^{1/4}(1-t)^{1/4}.dt$$

$$\int_0^1 4^{1/2}.t^{1/4}(1-t)^{1/4}.dt$$

$$\int_0^1 t^{1/4}(1-t)^{1/4}dt \Rightarrow 8\int_0^1 t^{5/4-1}(1-t)^{5/4-1}dt$$

$$= 8 \times B(5/4, 5/4) = 8\,\frac{\Gamma\frac{5}{4}\,\Gamma\frac{5}{4}}{\Gamma\frac{10}{4}}$$

$$= \frac{\cancel{8}^{2}\,\frac{1}{\cancel{4}}\Gamma\frac{1}{4}\cdot\frac{1}{\cancel{4}}\Gamma\frac{1}{4}}{3\frac{6}{4}\cdot\frac{\cancel{2}}{4}\cdot\Gamma\frac{1}{2}}$$

$$\int_0^3 x^{1/2}(27-x^3)^{-1/2}\,dx.$$

$x = 3t^{1/3}$

$x^2 = 9t^{2/3}$

$x^3 = 27t$

$3x^2\,dx = 27\,dt$

$3\times 9t^{2/3}\,dx = 27\,dt$

$dx = \dfrac{dt}{t^{2/3}}$

L.L $\Rightarrow x=0,\ t=0$

U.L $\Rightarrow x=3,\ t=1$

$$\int_0^1 \sqrt{3}\,t^{1/6}(27-27t)^{-1/2}\cdot\frac{dt}{t^{2/3}}$$

$$\int_0^1 \sqrt{3}\,t^{\frac16-\frac23}\frac{1}{\sqrt{27}}(1-t)^{-1/2}\cdot dt$$

$$\frac13\int_0^1 t^{\frac{-1}{2}}(1-t)^{-1/2}\cdot dt$$

$$=\frac13\int_0^1 t^{\frac12-1}(1-t)^{\frac12-1}\cdot dt$$

$$=\frac13 B\left(\frac12,\frac12\right)=\frac13\,\frac{\Gamma\frac12\,\Gamma\frac12}{\Gamma 1}=\frac13\sqrt{\pi}\sqrt{\pi}$$

$$=\frac{\pi}{3}.$$

$$\rightarrow \int_0^\infty \sqrt{x}\cdot e^{-\sqrt[6]{x}}\,dx.$$

$\sqrt[6]{x} = t$

$x = t^6$

$dx = 6t^5\,dt$

$$=\int_0^\infty t^3\cdot e^{-t}\cdot 6t^5\,dt$$

$$=6\int_0^\infty e^{-t}\cdot t^{8}\,dt \Rightarrow 6\int_0^\infty e^{-t}\cdot t^{9-1}\,dt \Rightarrow 6\,\Gamma 9.$$

$\Rightarrow 6\times 8!$

3, 6, 9, 15, 27 , N = 5

$$\frac{\Sigma X_i}{N} = \frac{3+6+9+15+27}{5}$$

$$= 12.$$

$$\sqrt{\frac{\Sigma (X_i - \mu)^2}{N}}$$

$$\sqrt{\frac{(3-12)^2+(6-12)^2+(9-12)^2+(15-12)^2+(27-12)^2}{5}}$$

$$= 6\sqrt{2} = 8.485$$

of Sample of size 3 that can be formed
population of size 5 without Replacement
3, 6, 9, 15, 27

$^5C_3 = 10.$

,9) (3, 6, 15) (3, 6, 27)

,15) (3, 9, 27)

5, 27)

15) (6, 9, 27)

5, 27)

, 15, 27)

6

9

10

Mean

S.D

⇒

$\sqrt{(}$

⇒

6 8 12
9 13 15
10 14 16
17

$x = 3t^{1/3}$
$x^2 = 9t^{2/3}$
$^3 = 27t$
$= 27\,dt$
$27\,dt$
$0 \to x=0, t=0$
$\to x=3, t=1$

Mean of S.D of Means

$$\mu_{\bar{X}} = \frac{\Sigma \bar{X}_i}{K} = 12$$

$$\boxed{\mu_{\bar{X}} = \mu}$$

S.D of sampling distribution of Means

$$\Rightarrow \sqrt{\frac{\Sigma(\bar{X}_i - \mu_{\bar{X}})^2}{K}}$$

$$\sqrt{\frac{(6-12)^2+(8-12)^2+(12-12)^2+(9-12)^2+(13-12)^2+(15-12)^2+(10-12)^2+(14-12)^2+(16-12)^2+(17-12)^2}{10}}$$

$$= 3.464$$

$$\boxed{\sigma_{\bar{X}} \neq \sigma}$$

$$\Rightarrow \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} \Rightarrow \frac{8.485}{\sqrt{3}}\sqrt{\frac{5-3}{5-1}}$$

$$= 3.4639$$

$$\simeq 3.464 = \sigma_{\bar{X}}$$

placement is $N^n = 5^2$

$= 25 = K.$

$$\left.\begin{matrix} (2,2) & (2,3) & (2,6) & (2,8) & (2,11) \\ (3,2) & (3,3) & (3,6) & (3,8) & (3,11) \\ (6,2) & (6,3) & (6,6) & (6,8) & (6,11) \\ (8,2) & (8,3) & (8,6) & (8,8) & (8,11) \\ (11,2) & (11,3) & (11,6) & (11,8) & (11,11) \end{matrix}\right\}$$ Sampling Distribution.

$$\left.\begin{matrix} 2 & 2.5 & 4 & 5 & 6.5 \\ 2.5 & 3 & 4.5 & 5.5 & 7 \\ 4 & 4.5 & 6 & 7 & 8.5 \\ 5 & 5.5 & 7 & 8 & 9.5 \\ 6.5 & 7 & 8.5 & 9.5 & 11 \end{matrix}\right\}$$ $\rightarrow \bar{x}_i$'s

S.D of Means

$$\mu_{\bar{x}} = \frac{\sum \bar{x}_i}{K} = \frac{150}{25} = 6$$

$$\boxed{\mu_{\bar{x}} = \mu = 6}$$

) of Sampling distribution of Means $\sigma_{\bar{x}} = \sqrt{\frac{\sum(\bar{x}_i - \mu_{\bar{x}})^2}{K}}$

$$\sqrt{\frac{[(2-6)^2 + (2.5-6)^2 + \dots\dots + (9.5-6)^2 + (11-6)^2]}{25}}$$

$$\sqrt{\frac{131}{25}} = 2.289.$$

$\sigma_{\bar{x}} = 2.32 \neq \sigma$

But $\frac{\sigma}{\sqrt{n}} = \frac{3.286}{\sqrt{2}} = 2.36$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 2.32}$$

i) Mean (ii, SD (iii, Mean of Sampling distribution of M...
iv) SD of Sampling distribution of Means and

y

A Population consists of 5 Numbers 3, 6, 9, 15, 27
out all the Possible Samples of Size 3 that can be
without Replacement from the finite Population
d Mean of the Population (iii, SD iv, Mean of the Sampling
bution of Mean iv SD of Sampling distribution of
ns and Verify

ven $N = 5$; $X_i \longrightarrow 2, 3, 6, 8, 11$

pulation Mean $\mu = \frac{\Sigma X_i}{N} = \frac{2+3+6+8+11}{5}$

$$= \frac{30}{5} = 6$$

$$.D = \sigma = \sqrt{\frac{\Sigma(X_i - \mu)^2}{N}}$$

$$= \sqrt{\frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}}$$

$$= \sqrt{\frac{16+9+4+25}{5}} = \sqrt{\frac{54}{5}}$$

$$= 3.286$$

IKT, no of Sample of size n that can be
ned from population of size N with

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S.D

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=

→ Find the Value of finite Population Correction factor for n = 10, N = 100.

→ When a Sample is taken from an Infinite Population what happens to the Standard error of Means if the Sample size is decreased from 800 to 200.

① → $\frac{N-n}{N-1}$

$= \frac{100-10}{100-1} = \frac{90}{99}$

$= 0.909$

$\simeq 0.91$

② → Given $n_1 = 800$ $n_2 = 200$

$\sigma_{\bar{x}_1} = \frac{\sigma}{\sqrt{n_1}} = \frac{\sigma}{\sqrt{800}} = \frac{\sigma}{20(\sqrt{2})}$ → ①

$\sigma_{\bar{x}_2} = \frac{\sigma}{\sqrt{n_2}} = \frac{\sigma}{\sqrt{200}} = \frac{\sigma}{10\sqrt{2}}$ → ②

from ① $\sigma_{\bar{x}_1} = \frac{1}{2}\left(\frac{\sigma}{10\sqrt{2}}\right)$

$= \frac{1}{2}\sigma_{\bar{x}_2}$

Standard error of Means got doubled when the Sample size decreased from 800 to 200.

→ A population Consists of 5 numbere 2, 3, 6, 8, 10, 11 consider all the possible Sample Size 2 which can be drawn with Replacement from this population

$$\sigma_{\bar{x}_1 \pm \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} \quad \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\sigma_{\bar{P}_1 \pm \bar{P}_2} = \sqrt{\sigma_{\bar{P}_1}^2 + \sigma_{\bar{P}_2}^2} = \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

tral limit Theorem:

Let $\bar{x}$ be the Mean of the Random Sample size n taken from a population having Mean and Variance σ^2 then

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ follows approximately

ndard Normal distribution

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad Z = \frac{(\bar{X}_1 \pm \bar{X}_2) - \mu_{\bar{x}_1 \pm \bar{x}_2}}{\sigma_{\bar{x}_1 \pm \bar{x}_2}}$$

$\bar{x}_1$ & $\bar{x}_2$ are Sample Means
μ_1 & μ_2 are Population Mean

$$Z = \frac{\bar{P} - P}{\sqrt{\frac{Pq}{n}}}$$

$$Z = \frac{(\bar{P}_1 - \bar{P}_2) - P_1 - P_2}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}}$$

$\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}} \neq \sigma$

ut $\sigma_{\bar{x}} = \begin{cases} \frac{\sigma}{\sqrt{n}} & \text{if pop is finite with Replacement or infinite pop with or without Replacement} \\ \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} & \text{if pop is finite with Replacement} \end{cases}$

here $\frac{N-n}{N-1}$ is called finite pop Correlation factor

ote: Omit finite pop Correction factor when N is infinite

portion: Let x Represents no of Successes out of trials then Observe proportion of Success or Sample Proportion is given by $\bar{p} = \frac{x}{n}$

$\bar{q} = 1 - \bar{p}$ $\qquad$ p - Population Proportion

$q = 1 - p$

opulation Proportion is denoted with p.

Mean of Sampling distribution of proportions $\mu_{\bar{p}} = p$

Standard deviation of Sampling distribution of portion $\sigma_{\bar{p}} = \sqrt{\frac{pq}{n}}$ (It is also Called Standard error of Proportion).

ampling distribution of differences & Sums for Means & roportions:

$\mu_{\bar{x}_1 \pm \bar{x}_2} = \mu_{\bar{x}_1} \pm \mu_{\bar{x}_2} = \mu_1 \pm \mu_2$.

$\mu_{\bar{p}_1 \pm \bar{p}_2} = \mu_{\bar{p}_1} \pm \mu_{\bar{p}_2} = p_1 \pm p_2$.

Prove that

$$\text{grad}(\bar{a}\cdot\bar{b}) = (\bar{b}\cdot\nabla)\bar{a} + (\bar{a}\cdot\nabla)\bar{b} + \bar{b}\times \text{Curl}\,\bar{a} + \bar{a}\times \text{Curl}(\bar{b})$$

$\bar{a}\times \text{Curl}(\bar{b})$

$$= \bar{a}\times(\nabla\times\bar{b})$$

$$\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$$

$$= \bar{a}\times\left[\sum\left(i\times\frac{\partial b}{\partial x}\right)\right]$$

$$= \sum\left[\bar{a}\times\left(i\times\frac{\partial b}{\partial x}\right)\right]$$

$$= \sum\left[\left(\bar{a}\cdot\frac{\partial b}{\partial x}\right)i - (\bar{a}\cdot i)\frac{\partial b}{\partial x}\right]$$

$$= \sum i\left(\bar{a}\cdot\frac{\partial b}{\partial x}\right) - \sum(\bar{a}\cdot i)\frac{\partial b}{\partial x}$$

$$= \sum i\left(\bar{a}\cdot\frac{\partial b}{\partial x}\right) - \left(\bar{a}\cdot\sum i\frac{\partial b}{\partial x}\right)b$$

$$= \sum i\left(\bar{a}\cdot\frac{\partial b}{\partial x}\right) - (\bar{a}\cdot\nabla)\bar{b} \longrightarrow (1)$$

lly

$$\bar{b}\times \text{Curl}\,\bar{a} = \sum i\left(\bar{b}\cdot\frac{\partial a}{\partial x}\right) - (\bar{b}\cdot\nabla)\bar{a} \longrightarrow (2)$$

(1) + (2) gives

$$\bar{a}\times \text{curl}\,\bar{b} + \bar{b}\times \text{Curl}\,\bar{a}$$

$$= \sum i\left(\bar{a}\cdot\frac{\partial b}{\partial x}\right) - (\bar{a}\cdot\nabla)\bar{b} + \sum i\left(\bar{b}\cdot\frac{\partial a}{\partial x}\right) - (\bar{b}\cdot\nabla)\bar{a}$$

$$\bar a \times \text{curl}\,\bar b + \bar b \times \text{curl}\,\bar a + (\bar a\cdot\nabla)\bar b + (\bar b\cdot\nabla)\bar a$$

$$= \sum i\left[\bar a\cdot\frac{\partial b}{\partial x} + \bar b\cdot\frac{\partial a}{\partial x}\right]$$

$$= \sum i\,\frac{\partial}{\partial x}(\bar a\cdot b)$$

$$= \nabla(\bar a\cdot\bar b). = R.H.S.$$

P.T

$$\text{div}(\bar a\times\bar b) = \bar b\cdot\text{curl}\,\bar a - \bar a\cdot\text{curl}\,\bar b$$

i.e $\nabla\cdot(\bar a\times\bar b) = b\cdot(\nabla\times\bar a) - \bar a\cdot(\nabla\times\bar b)$.

<u>L.H.S</u>

$$\nabla\cdot(\bar a\times\bar b)$$

$$= \sum i\cdot\frac{\partial}{\partial x}(\bar a\times\bar b)$$

$$= \sum i\cdot\left[\frac{\partial\bar a}{\partial x}\times\bar b + a\times\frac{\partial\bar b}{\partial x}\right]$$

$$= \sum i\left[\frac{\partial\bar a}{\partial x}\times b\right] + \sum i\left[\bar a\times\frac{\partial b}{\partial x}\right].$$

$$= \sum\left(i\times\frac{\partial a}{\partial x}\right)\cdot b - \sum i\cdot\left(\frac{\partial\bar b}{\partial x}\times\bar a\right)$$

$$= \bar b\cdot\sum\left(i\times\frac{\partial\bar a}{\partial x}\right) - \sum\left(i\times\frac{\partial\bar b}{\partial x}\right)\cdot\bar a$$

$$= \bar b\cdot\text{curl}\,\bar a - \bar a\cdot\text{curl}\,\bar b$$

$$= b\cdot(\nabla\times\bar a) - \bar a\cdot(\nabla\times\bar b)$$

$$= R.H.S$$

→ Prove th

$\nabla f\times\nabla$

We Kn

Curl $\bar a$

$\nabla\cdot(\bar a\times$

Put $\bar a$

L.H.S :

$= \nabla g\cdot(\nabla$

$= \nabla g.$ C

$= \nabla g.$

$= 0$

$\nabla\cdot(\nabla f\times$

div$(\nabla$

→ If f

functions

$+ \nabla f\cdot\nabla g$

div $(f\nabla g$

$\rightarrow \quad B(m,n) = a^m \cdot b^n \int_0^\infty \frac{x^{m-1}\,dx}{(ax+b)^{m+n}}$

Consider RHS.

$$\Rightarrow a^m \cdot b^n \int_0^\infty \frac{x^{m-1}\,dx}{(ax+b)^{m+n}}$$

$$= a^m \cdot b^n \cdot \int_0^\infty \frac{x^{m-1}\,dx}{\left[b\left[1+\frac{ax}{b}\right]\right]^{m+n}}$$

$$\frac{ax}{b} = t \Rightarrow dx = \frac{b}{a} \cdot dt$$

$$\frac{a^m \cdot b^n}{b^{m+n}} \int_0^\infty \frac{x^{m-1}\,dx}{\left(1+\frac{ax}{b}\right)^{m+n}}$$

$$a^m\, b^{\cancel{n}-m-\cancel{n}} \int_0^\infty \frac{\left(\frac{bt}{a}\right)^{m-1} \cdot \frac{b}{a} \cdot dt}{(1+t)^{m+n}}$$

$$a^m b^{-m} \int_0^\infty \frac{\frac{b^{m-\cancel{1}+\cancel{1}}}{a^{m-\cancel{1}+\cancel{1}}} \times t^{m-1}}{(1+t)^{m+n}}\,dt$$

$$\cancel{a^m}\, \cancel{b^{-m}} \cdot \frac{\cancel{b^m}}{\cancel{a^m}} \int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}}\,dt \qquad t = x \quad dt = dx$$

$$= \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = B(m,n)$$

(iii) Form

$\rightarrow \int_b^a (x-b)^{m-1}(a-x)^{n-1}\,dx = B(m,n)\,(a-b)^{m+n-1}$

W.K.T

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx.$$

Put $x = \dfrac{t-b}{a-b}$ $\quad dx = \dfrac{dt}{a-b}.$ $\quad$ $UL \Rightarrow 1 = \dfrac{t-b}{a-b}$, $t=a$; $\quad LL \Rightarrow 0 = \dfrac{t-b}{a-b}$, $t=b$.

$$\int_b^a \left(\frac{t-b}{a-b}\right)^{m-1}\left(1-\left(\frac{t-b}{a-b}\right)\right)^{n-1}\cdot\frac{dt}{a-b}$$

$$\int_b^a \left(\frac{t-b}{a-b}\right)^{m-1}\left(\frac{a-\cancel{b}-t+\cancel{b}}{a-b}\right)^{n-1}\frac{dt}{a-b}$$

$$\Rightarrow \int_b^a \frac{(t-b)^{m-1}(a-t)^{n-1}\,dt}{(a-b)^{m-1+n-\cancel{1}+\cancel{1}}}$$

$$\Rightarrow \int_b^a \frac{(t-b)^{m-1}(a-t)^{n-1}\,dt}{(a-b)^{m+n-1}} = B(m,n).$$

$$\int_b^a (t-b)^{m-1}(a-t)^{n-1}\,dt = B(m,n)\cdot(a-b)^{m+n-1}.$$

$$\int_5^7 (x-5)^6 (7-x)^3 dx = 2^{10} B(7,4) \longrightarrow ①$$

in the 5^{th} form.

$$\int_b^a (x-b)^{m-1} (a-x)^{n-1} dx = B(m,n)(a-b)^{m+n-1} \longrightarrow ②$$

from ① & ②

$b = 5,\ a = 7,\ m = 7,\ n = 4.$

S.T $$\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} B(m+1, n+1)$$

S.T $$\int_0^\infty \frac{x^{m-1} dx}{(x+a)^{m+n}} = a^{-n} B(m,n).$$

amma Function

We Know that

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx, \; n>0$$

Properties of Gamma Function:

$$\Gamma 1 = \int_0^\infty e^{-x}\, dx.$$

$$= \left(-e^{-x}\right)_0^\infty = -e^{-\infty} + e^{-0}$$

$$= 1$$

$$\boxed{\Gamma 1 = 1}$$

S.T $\Gamma n = (n-1)\,\Gamma n-1, \; n>1.$

W.K.T.

$$\Rightarrow \int e^{-x} x^{n-1}\, dx = (n-1) \int e^{-x} x^{n-2}\, dx.$$

$$\text{LS} = \left[-x^{n-1} e^{-x}\right]_0^\infty + \int_0^\infty \left[(n-1)x^{n-2} e^{-x}\right]$$

$$= (n-1) \int_0^\infty x^{(n-1)-1} \cdot e^{-x}\, dx.$$

$$= (n-1)\,\Gamma n-1$$

lly $\Gamma n+1 = n\,\Gamma n.$

If n is a Positive fraction

$$\Gamma n = (n-1)(n-2)(n-3)\cdots(n-r)\Gamma n-r$$

where $n-r>0$

$$\Gamma \tfrac{9}{2} = \left(\tfrac{9}{2}-1\right)\left(\tfrac{9}{2}-2\right)\left(\tfrac{9}{2}-3\right)\left(\tfrac{9}{2}-4\right)\Gamma\left(\tfrac{9}{2}-4\right)$$

$$\Gamma \tfrac{9}{2} = \tfrac{7}{2}\times\tfrac{5}{2}\times\tfrac{3}{2}\times\tfrac{1}{2}\,\Gamma\tfrac{1}{2}$$

$$\Gamma\tfrac{13}{3} = \left(\tfrac{13}{3}-1\right)\left(\tfrac{13}{3}-2\right)\left(\tfrac{13}{3}-3\right)\left(\tfrac{13}{3}-4\right)\Gamma\tfrac{13}{3}-4$$

$$= \tfrac{10}{3}\cdot\tfrac{7}{3}\cdot\tfrac{4}{3}\cdot\tfrac{1}{3}\,\Gamma\tfrac{1}{3}$$

If n is a non-negative Integer $\Gamma n+1 = n!$

$$= \Gamma n+1 = n\Gamma n$$

$$= n(n-1)(n-2)\Gamma n-2$$

$$= n(n-1)(n-2)(n-3)\cdots 1\Gamma 1$$

$$= n!$$

Relation b/w B, Gamma Function:

$$B(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n} \quad \text{where } m>0,\ n>0$$

Compute $\Gamma\frac{11}{2}$

$$\Gamma\tfrac{11}{2} = \left(\tfrac{11}{2}-1\right)\left(\tfrac{11}{2}-2\right)\left(\tfrac{11}{2}-3\right)\left(\tfrac{11}{2}-4\right)\left(\tfrac{11}{2}-5\right)\Gamma\left(\tfrac{11}{2}-5\right)$$

$$= \tfrac{9}{2}\cdot\tfrac{7}{2}\cdot\tfrac{5}{2}\cdot\tfrac{3}{2}\cdot\tfrac{1}{2}\,\Gamma\tfrac{1}{2}$$

& T

w.k.

p

S.T $\Gamma(1/2) = \sqrt{\pi}$

W.K.T

$$B(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$$

put $m = n = 1/2$

$$B(1/2, 1/2) = \frac{\Gamma 1/2 \, \Gamma 1/2}{\Gamma 1/2 + 1/2} = \left(\Gamma 1/2\right)^2 \longrightarrow (1)$$

$$\therefore B(1/2, 1/2) = \int_0^1 x^{1/2 - 1} (1-x)^{1/2 - 1} dx.$$

$$= \int_0^1 x^{-1/2} (1-x)^{-1/2} dx$$

$x = \sin^2\theta$, $1 - x = \cos^2\theta$ U.L = $\theta = \pi/2$
$dx = 2 \sin\theta \cos\theta \, d\theta$ L.L = $\theta = 0$.

$$= \int_0^{\pi/2} \frac{1}{\cancel{\sin\theta}} \cdot \frac{1}{\cancel{\cos\theta}} \cdot 2 \cancel{\sin\theta} \cancel{\cos\theta} \, d\theta$$

$$= 2 \int_0^{\pi/2} d\theta$$

$$= 2 [\theta]_0^{\pi/2} = 2 [\pi/2 - 0] = 2\left(\frac{\pi}{2}\right)$$

$$= \pi.$$

$$B(1/2, 1/2) = \pi$$

$$\left(\Gamma 1/2\right)^2 = \pi$$

$$\boxed{\Gamma 1/2 = \sqrt{\pi}}$$

$\Rightarrow$ S.T $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

L.H.S $= \int_0^\infty e^{-x^2} dx$.

Put $x^2 = t$

$2x\, dx = dt$

$dx = \frac{dt}{2x} = \frac{dt}{2\sqrt{t}} = \frac{1}{2} t^{-1/2} dt$

$\Rightarrow \int_0^\infty e^{-t} \frac{dt}{2\sqrt{t}}$

$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^{-1/2} dt$ $\qquad \Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$.

$= \frac{1}{2} \Gamma \frac{1}{2} = \frac{1}{2}\sqrt{\pi}$

$= \frac{\sqrt{\pi}}{2}$.

Evaluate

$\int_0^1 \frac{35 x^3}{32(\sqrt{1-x})} dx$

$1 - x = t$ $\qquad$ U.L $= 0 = t$

$-dx = dt$ $\qquad$ L.L $= t = 1$.

$dx = -dt$

$\frac{35}{32} \int_1^0 \frac{(1-t)^3}{\sqrt{t}} \times -dt$

$\frac{35}{32} \int_0^1 (1-t)^{2+1} t^{-1/2} dt$

$$= \frac{35}{32}\int_0^1 t^{1/2-1}(1-t)^{4-1}\,dt$$

$t = x,\ dt = dx.$

$$\frac{35}{32}\int_0^1 x^{1/2-1}(1-x)^{4-1}\,dx.$$

$$\frac{35}{32} \times B(1/2, 4).$$

$$\frac{35}{32} \times \frac{\Gamma 1/2\ \Gamma 4}{\Gamma 9/2}$$

$$= \frac{35}{32} \times \frac{\sqrt{\pi} \times 3!}{\Gamma 9/2}$$

$$= \frac{35}{32} \times \frac{6\cancel{\sqrt{\pi}}}{\frac{7\times5\times3}{16}\cancel{\sqrt{\pi}}} = 1$$

$$\int_0^1 x^4\left(\log\frac{1}{x}\right)^3 dx.$$

$\log 1/x = t$

$1/x = e^{t}$

$\boxed{x = e^{-t}}.$

$dx = -e^{-t}dt$

$U.L = \log 1 = t$

$t = 0$

$L.L = \log\frac{1}{0} = t$

$t = \infty$

$$= \int_\infty^0 e^{-4t}\, t^3 \times -e^{-t}\,dt$$

$$= \int_0^\infty e^{-5t} t^3\,dt$$

$5t = u$

$5\,dt = du$

$dt = \frac{du}{5}$

$$= \int_0^\infty e^{-u}\cdot\frac{u^3}{125}\cdot\frac{du}{5}$$

$$\frac{1}{625}\int^\infty e^{-u}u^3\,du$$

$\int x^4 \left[\log\left(\frac{1}{x}\right)\right]^3 dx$

$\frac{1}{625}\int_0^\infty e^{-u}\cdot u^{4-1}du.$

$\Gamma n = \int_0^\infty e^{-x}\cdot x^{n-1}dx.$

$\frac{1}{625}\Gamma 4 = \frac{1}{625}\times 3!$

$= \frac{6}{625}$

$x^2(\log 1/x)^3 dx$

$\log 1/x = t$

$e^t = 1/x$

$x = 1/e^t$

$x = e^{-t}$

$dx = -e^{-t}dt$

$\int e^{-2t}t^3(-e^{-t}dt)$

$e^{-3t}\cdot t^3\cdot dt$

$U\cdot L = 0$

$L\cdot L = \infty$

$3t = u \Rightarrow t = u/3$

$3dt = du$

$dt = \frac{du}{3}$

$e^{-u}\ \frac{u^3}{27}\cdot\frac{du}{3}$

$\frac{1}{81}\int_0^\infty e^{-u}u^3du.$

$= \frac{1}{81}\int_0^\infty e^{-u}u^{4-1}du.$

$\Rightarrow \int_0^\infty 3$

$= \frac{3}{4}\int_0$

$= \frac{1}{81} \Gamma 4$

$= \frac{1}{81} \times 3!$

$= 6/81 = 2/27$

$\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$

$\int_0^\infty 3^{-4x^2} dx$

$3 = e^{\log_e 3}$

$4x^2 = t$

$4(2x) dx = dt$

$3^{-4x^2} = e^{-4x^2 \log 3}$

$= e^{-(2x\sqrt{\log 3})^2}$

Put $y = 2x\sqrt{\log 3}$

$dy = 2\sqrt{\log 3} \cdot dx$

$\therefore \int_0^\infty e^{-y^2} \times \frac{dy}{2\sqrt{\log 3}}$

$= \frac{1}{2\sqrt{\log 3}} \int_0^\infty e^{-y^2} dy$

$= \frac{1}{2\sqrt{\log 3}} \times \frac{\sqrt{\pi}}{2}$

$$\int_0^\infty a^{-bx^2}\,dx.$$

$$a = e^{\log_e a}$$

$$a^{-bx^2} = \left(e^{\log_e a}\right)^{-bx^2}$$

S.T $\int_0^\infty \sqrt{x}\cdot e^{-x^3}\,dx$

$x^3 = t$

$3x^2\cdot dx = dt$

$dx = \frac{dt}{3x^2}$

$$= \int_0^\infty e^{-t}\cdot\frac{dt}{3x^2}\cdot\sqrt{x}$$

$$= \int_0^\infty e^{-t}\cdot\frac{dt}{3x^{3/2}} \Rightarrow \int_0^\infty e^{-t}\cdot\frac{dt}{3\sqrt{t}}$$

$$\Rightarrow \frac{1}{3}\int_0^\infty \frac{e^{-t}}{\sqrt{t}}\,dt.$$

$$= \frac{1}{3}\int_0^\infty e^{-t}\cdot t^{-1/2}\,dt$$

$$= 1/3\,\Gamma 1/2 = 1/3\sqrt{\pi}$$

$\int_0^1$

$\int_0^\infty$

=

$$\int_0^1 \frac{1}{\sqrt{-\log x}}\,dx.$$

$-\log x = t \Rightarrow \log 1/x = t$

$-\frac{1}{x}dx = dt \qquad 1/x = e^t$

$\qquad\qquad x = e^{-t}$

$dx = \cancel{x}\,dt$

$$\int_\infty^0 \frac{-e^{-t}}{\sqrt{t}}\,dt$$

$$\Rightarrow \int_0^\infty \frac{e^{-t}}{\sqrt{t}}\,dt$$

$$= \int_0^\infty e^{-t}\cdot t^{-1/2}\,dt \Rightarrow \int_0^\infty e^{-t}\,t^{1/2-1}\,dt$$

$$= \Gamma(1/2) = \sqrt{\pi}$$

$$\int_0^1 x^7(1-x)^5\,dx.$$

$$\Rightarrow \int_0^1 x^{8-1}(1-x)^{6-1}\,dx.$$

$$= B(8,6)$$

$$= \frac{\Gamma 8\,\Gamma 6}{\Gamma 14} = \frac{7!\,5!}{13!} =$$

$$\int_0^{\pi/2} \sin^{7/2}\theta \cdot \cos^{3/2}\theta \, d\theta = 9.$$

W.K.T

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m,n).$$

$$2m-1 = 7/2 \qquad 2n-1 = 3/2$$
$$2m = 9/2 \qquad 2n = 5/2$$
$$m = 9/4 \qquad n = 5/4$$

$$\Rightarrow \frac{1}{2} B\left(\frac{9}{4}, \frac{5}{4}\right).$$

$$= \frac{1}{2} \frac{\Gamma 9/4 \, \Gamma 5/4}{\Gamma 14/4}$$

$$= \frac{1}{2} \frac{\left(\frac{9}{4}-1\right)\left(\frac{9}{4}-2\right)\Gamma\left(\frac{9}{4}-2\right) \cdot \left(\frac{5}{4}-1\right)\Gamma\left(\frac{5}{4}-1\right)}{\left(\frac{14}{4}-1\right)\left(\frac{14}{4}-2\right)\left(\frac{14}{4}-3\right)\Gamma\left(\frac{14}{4}-3\right)}$$

$$= \frac{1}{2} \cdot \frac{\frac{5}{4} \cdot \frac{1}{4} \Gamma \frac{1}{4} \cdot \frac{1}{4} \Gamma 1/4}{\frac{10}{4} \cdot \frac{6}{4} \cdot \frac{2}{4} \Gamma 1/2}$$

$\rightarrow \int_0^1 x^{5/2}(1-x^2)^{3/2}dx$

$\rightarrow \int_0^2 x(8-x^3)^{1/3}dx$

$x^3 = 8y$

$3x^2 dx = 8\,dy$

$dx = \frac{8}{3x^2}dy$

$x = (8y)^{1/3} = 2y^{1/3}$

$dx = \frac{\cancel{8}\,2}{3 \times \cancel{4} \times y^{2/3}}dy$

$dx = \frac{2}{3y^{2/3}}dy$

$= \int_0^1 \cancel{2}y^{1/3}(8-8y)^{1/3} \cdot \frac{\cancel{8}\,4}{\cancel{12}\,\cancel{6}\,3\,y^{2/3}}dy$

$= \frac{8}{3}\int_0^1 x^{-1/3}(1-x)^{1/3}dx$

Put $y = x$
$dy = dx$

$= \frac{8}{3}\int_0^1 x^{2/3-1}(1-x)^{4/3-1}dx$

$= \int_0^1 x^{m-1}(1-x)^{n-1}dx$

$U.L = 1$
$L.L = 0$

$\Rightarrow \frac{8}{3}B\left(\frac{2}{3},\frac{4}{3}\right) = \frac{8}{3}\cdot\frac{\Gamma\frac{2}{3}\,\Gamma\frac{4}{3}}{\Gamma 2} \Rightarrow \frac{8}{3}\cdot\frac{\Gamma\frac{2}{3}\cdot\frac{1}{3}\Gamma\frac{1}{3}}{1!}$

$= \frac{8}{9}\Gamma\frac{2}{3}\,\Gamma\frac{1}{3}$

$\rightarrow \int_0^1 x^{5/2}(1-x^2)^{3/2}dx$

$x^2 = y$

$2x\,dx = dy$

$dx = \frac{1}{2x}dy$

$dx = \frac{1}{2\sqrt{y}}dy$

$\int_0^1 (\sqrt{y})^{5/2}(1-y)^{3/2}\cdot\frac{1}{2\sqrt{y}}dy$

$= \frac{1}{2}\int_0^1 \frac{y^{5/4}}{y^{1/2}}(1-y)^{3/2}\cdot dy \Rightarrow \frac{1}{2}\int_0^1 y^{3/4}(1-y)^{3/2}dy$

$\Rightarrow \frac{1}{2}\int y^{7/4-1}(1-y)^{5/2-1}dy$

$= \frac{1}{2}B\left(\frac{7}{4},\frac{5}{2}\right) = \frac{1}{2}\frac{\Gamma\frac{7}{4}\,\Gamma\frac{5}{2}}{\Gamma\frac{17}{4}}$

$$\frac{1}{2}\left[\frac{\left(\frac{7}{4}-1\right)\left(\frac{7}{4}-1\right)\quad\left(\frac{5}{2}-1\right)\left(\frac{5}{2}-2\right)\Gamma\left(\frac{5}{2}-2\right)}{\left(\frac{17}{4}-1\right)\left(\frac{17}{4}-2\right)\left(\frac{17}{4}-3\right)\left(\frac{17}{4}-4\right)\Gamma\left(\frac{17}{4}-4\right)}\right]$$

$$\frac{1}{2}\left[\frac{\left(\frac{3}{4}\right)\Gamma\frac{3}{4}\quad\frac{3}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}}{\frac{13}{4}\cdot\frac{9}{4}\cdot\frac{5}{4}\cdot\frac{1}{4}\Gamma\frac{1}{4}}\right]$$

$$\frac{\frac{1}{\cancel{2}}\cdot\frac{\cancel{3}}{4}\cdot\frac{\cancel{3}}{\cancel{4}}\Gamma\frac{3}{4}\cdot\sqrt{\pi}}{\frac{1}{\cancel{4}}\cdot\frac{5}{\cancel{4}_2}\cdot\frac{9}{\cancel{4}}\cdot\frac{13}{4}\Gamma\frac{1}{4}}$$

$$= \frac{\sqrt{\pi}}{2}\cdot\frac{8\sqrt{\pi}}{65\,\Gamma\frac{1}{4}}$$

$$\frac{1}{(1-x^3)^{1/3}}\,dx.$$

$x^3 = y \qquad x = y^{1/3}$

$3x^2 dx = dy$

$dx = \frac{1}{3x^2}dy.$

$= \frac{1}{3y^{2/3}}dy$

$$\frac{1}{(1-y)^{1/3}}\cdot\frac{1}{3y^{2/3}}\,dy.$$

$$\int_0^1 \frac{1}{y^{2/3}(1-y)^{1/3}}\,dy.$$

$$\frac{1}{3}\int_0^1 y^{-2/3}(1-y)^{-1/3}\,dy$$

$$\frac{1}{3}\int_0^1 y^{1/3-1}(1-y)^{2/3-1}\,dy.$$

$$= \frac{1}{3} B\left(\frac{1}{3},\frac{2}{3}\right)$$

$$= \frac{1}{3}\,\frac{\Gamma\frac{1}{3}\,\Gamma\frac{2}{3}}{\Gamma 1}$$

$$=$$

$$\rightarrow \int_0^1 x^3\sqrt{1-x}\,dx.$$

$$\Rightarrow \int_0^1 x^3(1-x)^{1/2}\,dx$$

$$\int_0^1 x^{4-1}(1-x)^{3/2-1}\,dx$$

$$= B\left(4,\frac{3}{2}\right). = \frac{\Gamma 4\,\Gamma\frac{3}{2}}{\Gamma\frac{11}{2}}$$

$$= \frac{3!\times\frac{1}{2}\Gamma\frac{1}{2}}{\left(\frac{11}{2}-1\right)\left(\frac{11}{2}-2\right)\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-4\right)\left(\frac{11}{2}-5\right)\Gamma\frac{1}{2}}$$

$$= \frac{6\cdot 3\sqrt{\pi}}{\frac{9}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\Gamma\frac{1}{2}}$$

$$= \frac{32}{315}$$

$$4\int_0^{\infty} \frac{x^2}{1+x^4}\,dx$$

$x^2 = t$

$2x\,dx = dt$

$dx = \frac{1}{2x}\,dt$

$dx = \frac{1}{2\sqrt{t}}\,dt$

$$4^2\int_0^{\infty} \frac{t}{1+t^2}\cdot\frac{1}{\cancel{2}\sqrt{t}}\,dt$$

$$2\int_0^{\infty} \frac{t^{3/2}}{1+t^2}\,dt$$

$$= 2\int_0^{\infty} t^{3/2}\,(1+t^2)^{-1}\,dt$$

$$= 2$$

$$\int_0^1 \frac{1\,dx}{\sqrt{-\log x}}$$

$-\log x = t \Rightarrow x^{-1} = e^{t}$

$x = e^{-t}$

$-\frac{1}{x}\,dx = dt$

$dx = -e^{-t}\,dt$

$U.L = t = -1$

$L.L = t = 0$

$$= \int_{-\infty}^{0} \frac{-e^{-t}\,dt}{\sqrt{t}}$$

$$= \int_0^{\infty} \frac{e^{-t}\,dt}{\sqrt{t}}$$

$n-1 = -1/2$

$n = 1/2$

$$= \int_0^{\infty} e^{-t}\,t^{-1/2}\,dt \Rightarrow \int_0^{\infty} e^{-x}\,x^{n-1}\,dx$$

$$= \int_0^{\infty} e^{-t}\,t^{1/2-1}\,dt = \Gamma\tfrac{1}{2} = \sqrt{\pi}$$

$\rightarrow \displaystyle\int_0^1 x^m (\log x)^n\, dx = \frac{(-1)^n \cdot n!}{(m+1)^{n+1}}$, where n is a +ve integer & $m > -1$

$$LHS = \int_0^1 x^m (\log x)^n\, dx$$

$\log x = -t$

$x = e^{-t}$

$dx = -e^{-t}\, dt$

$U.L = 0,\ L.L = \infty$

$$= -\int_\infty^0 (e^{-t})^m (-t)^n e^{-t}\, dt$$

$$= \int_0^\infty e^{-tm} (-t)^n e^{-t}\, dt$$

$$\boxed{\int_0^\infty e^{-kx} x^{n-1}\, dx = \frac{\Gamma n}{k^n}}$$

$$= \int_0^\infty e^{-t(m+1)} (-t)^{(n+1)-1}\, dt$$

$$= \cancel{\frac{\Gamma m+1}{}} \quad \frac{\Gamma n+1}{(m+1)^{(n+1)}}$$

$$= \frac{(-1)^n \cdot n!}{(m+1)^{n+1}}.$$

$$\rightarrow \int_0^\infty \frac{x^4 \cdot (1+x^5)\, dx}{(1+x)^{15}}$$

$$= \int_0^\infty \frac{x^4}{(1+x)^{15}} + \int_0^\infty \frac{x^9\, dx}{(1+x)^{15}}$$

$$= \int_0^\infty \frac{x^{5-1}\, dx}{(1+x)^{5+10}} + \int_0^\infty \frac{x^{10-1}\, dx}{(1+x)^{5+10}} = B(5,10) + B(10,5)$$

$$= \frac{\Gamma 5\, \Gamma 10}{\Gamma 15} + \frac{\Gamma 10\, \Gamma 5}{\Gamma 15}$$

$$= \frac{4!\, 9!}{14!} + \frac{9!\, 4!}{14!} = \frac{2 \times 4! \times 9!}{}$$

$$= \frac{1}{13 \times 7 \times 11 \times 5}$$

$$= \frac{1}{5005}$$

$$\int_0^1 x^m (1-x^n)^p \, dx$$

$x^n = t \Rightarrow x = \sqrt[n]{t}$

$n x^{n-1} dx = dt$

$dx = \frac{1}{n x^{n-1}} dt$

$= \frac{x}{n x^n} dt$

$= \frac{x}{nt} dt$

$= \frac{(t)^{1/n}}{nt} dt$

$$\int_0^1 \left((t)^{1/n}\right)^m (1-t)^p \cdot \frac{t^{1/n}}{nt} dt$$

$$\frac{1}{n}\int_0^1 \frac{t^{m/n + 1/n}}{t} (1-t)^p \, dt$$

$$\frac{1}{n}\int_0^1 t^{\frac{m+1}{n}-1} (1-t)^p \, dt$$

$$\frac{1}{n}\int_0^1 t^{\frac{(m-n)+1}{n}} (1-t)^p \, dt$$

$m - 1 = \frac{m - n + 1}{n}$

$nm - n = m - n + 1$

$nm = m + 1$

$m(n-1) = 1$

$m = \frac{1}{n-1}$

$$\frac{1}{n}\int_0^1 t^{\frac{m+1}{n}-1} (1-t)^{(p+1)-1} \, dt$$

$$= \frac{1}{n} B\left(\frac{m+1}{n}, p+1\right) = \frac{1}{n} \frac{\Gamma\left(\frac{m+1}{n}\right)\Gamma(p+1)}{\Gamma\left(\frac{m+1}{n}+p+1\right)}$$

$\frac{m-n+1}{n} + 1 - 1$

$\frac{m-\cancel{n}+1+\cancel{n}}{n} - 1$

$\Rightarrow \frac{m+1}{n} - 1$

→ Prove that $\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^4}}$

$x^2 = \sin\theta$

$2x \cdot dx = \cos\theta\, d\theta$

$dx = \frac{\cos\theta}{2\sqrt{\sin\theta}}\, d\theta$

L.L $= \theta = 0$

U.L: $\theta = \pi/2$

$$= \int_0^{\pi/2} \frac{\sin\theta \cdot \frac{\cos\theta}{2\sqrt{\sin\theta}}}{\cancel{\cos\theta}} \cdot d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$$

$$\int_0^{\pi/2} \sin^p x \cos^q x\, dx = \frac{\frac{1}{2}\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}}$$

$$\frac{1}{2}\int_0^{\pi/2} \sin^{1/2}\theta \cdot \cos^0\theta\, d\theta$$

$$= \frac{1}{2}\left[\frac{1/2 \cdot \Gamma\frac{3}{4}\,\Gamma\frac{1}{2}}{\Gamma\frac{5}{4}}\right] = \frac{1}{2}\left[\frac{\cancel{\frac{1}{2}} \cdot \Gamma\frac{3}{4} \cdot \sqrt{\pi}}{\cancel{\frac{1}{4}}\,\Gamma\frac{1}{4}}\right]$$

$$= \frac{\sqrt{\pi}\,\Gamma\frac{3}{4}}{\Gamma\frac{1}{4}} \longrightarrow (i)$$

→ $\int_0^1 \frac{dx}{\sqrt{1+x^4}}$

$x^2 = \tan\theta$

$2x\, dx = \sec^2\theta\, d\theta$

$dx = \frac{\sec^2\theta}{2\sqrt{\tan\theta}}\, d\theta$

U.L $= \pi/4$

L.L $= 0$

$$= \int_0^{\pi/4} \frac{\sec^2\theta}{2\sqrt{\tan\theta} \cdot \cancel{\sec\theta}} \cdot d\theta$$

$$\frac{1}{2}\int_0^{\pi/24} \frac{\sec\theta}{\sqrt{\tan\theta}} \cdot d\theta$$

$$\frac{1}{2}\int_0^{\pi/4} \frac{1}{\cos\theta} \times \frac{\sqrt{\cos\theta}}{\sqrt{\sin\theta}} \cdot d\theta$$

$$\frac{1}{2}\int_0^{\pi/24} \sin^{-1/2}\theta \cdot \cos^{-1/2}\theta \cdot d\theta$$

$$= \frac{1}{2}\left[\frac{1}{2}\,\frac{\Gamma\frac{-\frac{1}{2}+1}{2}\,\Gamma\frac{-\frac{1}{2}+1}{2}}{\Gamma\frac{-\frac{1}{2}+-\frac{1}{2}+2}{2}}\right]$$

$$= \frac{1}{4}\left[\frac{\Gamma\frac{1}{4}\,\Gamma\frac{1}{4}}{\Gamma\frac{1}{2}}\right] = \frac{1}{4\sqrt{\pi}}\left[\Gamma\frac{1}{4}\,\Gamma\frac{1}{4}\right]$$

$$\frac{1}{2}\int_0^{\pi/4} \frac{2}{\sqrt{2\sin\theta\cos\theta}}\, d\theta$$

$$= \frac{1}{\sqrt{2}}\int_0^{\pi/4} \frac{1}{\sqrt{\sin 2\theta}}\, d\theta \Rightarrow \frac{1}{\sqrt{2}}\int_0^{\pi/2} \frac{1}{\sqrt{\sin u}} \cdot \frac{du}{2}$$

$2\theta = u$

$2d\theta = du$

$U.L = \pi/2$

$L.L = 0$

$$\Rightarrow \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \sin^{-1/2} u \cos^0 u\, du.$$

=

=

=

(1)

=

=

$\frac{2}{n}$

$$= \frac{1}{2\sqrt{2}} \int_0^{\pi/2} \sin^{1/2} u \cos^0 u \, du$$

$$= \frac{1}{2\sqrt{2}} \left[\frac{\frac{1}{2} \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{4}\right)} \right]$$

$$= \frac{\sqrt{\pi}}{4\sqrt{2}} \left[\frac{\Gamma(1/4)}{\Gamma(3/4)} \right] \longrightarrow (2)$$

(1) × (2)

$$= \frac{\sqrt{\pi} \cdot \cancel{\Gamma\left(\frac{3}{4}\right)}}{\cancel{\Gamma(1/4)}} \times \frac{\sqrt{\pi}}{4\sqrt{2}} \left[\frac{\cancel{\Gamma(1/4)}}{\cancel{\Gamma(3/4)}} \right]$$

$$= \frac{\pi}{4\sqrt{2}}$$

$$\rightarrow \int_0^1 \frac{1}{\sqrt{1-x^n}} \, dx$$

$x^n = \sin^2\theta$ $\qquad x^n = t$

$n x^{n-1} dx = 2 \sin\theta \cos\theta \, d\theta$ $\qquad n x^{n-1} dx = dt$

$\frac{n \cdot \sin^2\theta}{x} dx = 2 \cancel{\sin\theta} \cos\theta \, d\theta$ $\qquad \frac{n}{x} \cdot t \cdot dx = dt$

$\frac{n \sin\theta}{\sin^{2/n}\theta} dx = 2\cos\theta \, d\theta$ $\qquad \frac{nt}{(t)^{1/n}} dx = dt$

$dx = \frac{\sin^{2/n}\theta \times 2\cos\theta}{n \sin\theta} d\theta$ $\qquad dx = \frac{(t)^{1/n}}{nt} dt$

$$= \int_0^{\pi/2} \frac{\sin^{2/n}\theta \cdot d\theta \times 2 \cos\theta}{n \sin\theta \cdot \cos\theta}$$

$$\frac{2}{n} \int_0^{\pi/2} \sin^{2/n - 1}\theta \, d\theta$$

$$\Rightarrow \frac{2}{n} \left[\frac{1}{2} \Gamma\left(\frac{1}{n}\right) \Gamma\left(\frac{1}{2}\right) \right.$$

$$\frac{\sqrt{\pi}}{n}\left[\frac{\Gamma(1/n)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}\right]$$

$$\int_0^\infty e^{-y^{1/m}}\,dy = m\Gamma m$$

$y^{1/m} = t \Rightarrow y = t^m$

$\log y^{1/m} = \log t$

$\frac{1}{m}\log y = \log t$

$\frac{1}{m}\cdot\frac{1}{y}\,dy = \frac{1}{t}\cdot dt$

$dy = t^{m-1}\cdot m\;dt$

$$\int_0^\infty e^{-t}\cdot t^{m-1}\cdot m\cdot dt$$

$$= m\int_0^\infty e^{-t}\cdot t^{m-1}\cdot dt$$

$$= m\Gamma m$$

$$\int_0^\infty e^{-x^4}\,dx$$

$x^4 = t \Rightarrow x = (t)^{1/4}$

$4x^3\,dx = dt$

$dx = \frac{dt}{4t^{3/4}}$

$$\int_0^\infty e^{-t}\cdot\frac{dt}{4t^{3/4}}$$

$$\frac{1}{4}\int_0^\infty e^{-t}\cdot t^{-3/4}\,dt \Rightarrow \frac{1}{4}\int_0^\infty e^{-t}\cdot t^{1/4-1}\,dt$$

$$\Rightarrow \frac{1}{4}\Gamma\frac{1}{4}.$$

No

Note : $\Gamma n \, \Gamma 1-n = \dfrac{\pi}{\sin n\pi}$.

$\rightarrow \displaystyle\int_0^{\pi/2} \tan^n\theta \, d\theta = \frac{\pi}{2} \cdot \sec \frac{n\pi}{2}$ where $|n| < 1$

$$= \int_0^{\pi/2} \frac{\sin^n\theta}{\cos^n\theta} \cdot d\theta$$

$$= \int_0^{\pi/2} \sin^n\theta \cdot \cos^{-n}\theta \cdot d\theta$$

$$= \frac{\frac{1}{2}\Gamma\frac{n+1}{2}\,\Gamma\frac{-n+1}{2}}{\Gamma\frac{n+(-n)+2}{2}} = \frac{1}{2}\Gamma\frac{n+1}{2}\,\Gamma\frac{1-n}{2}$$

$$= \frac{1}{2}\Gamma\frac{n+1}{2}\,\Gamma 1-\left(\frac{n+1}{2}\right)$$

$$= \frac{1}{2} \cdot \frac{\pi}{\sin\left(\frac{n+1}{2}\right)\pi}$$

$$= \frac{\pi}{2} \cdot \frac{1}{\sin\left(\frac{n\pi}{2} + \frac{\pi}{2}\right)}.$$

$$= \frac{\pi}{2} \cdot \frac{1}{\cos\frac{n\pi}{2}} = \frac{\pi}{2} \sec \frac{n\pi}{2}$$

$\Rightarrow \displaystyle\int_0^2 x^3\sqrt{2-x}\, dx$

$x = 2y$

$dx = 2\,dy$

$$= \int_0^1 8y^3\sqrt{2-2y} \cdot 2\,dy.$$

$$\int_0^{R} y^3 \sqrt{2-2y}\, dy$$

$$16 \times \sqrt{2} \int_0^1 y^3 (1-y)^{1/2}\, dy \qquad \int_0^1 x^{m-1} (1-x)$$

$$= 16\sqrt{2} \int_0^1 y^{4-1} (1-y)^{\frac{3}{2}-1}\, dy$$

$$= 16\sqrt{2}\; B\left(4, \tfrac{3}{2}\right)$$

$$= 16\sqrt{2} \left[\frac{\Gamma 4\, \Gamma \frac{3}{2}}{\Gamma \frac{11}{2}} \right]$$

$$= 16\sqrt{2} \left[\frac{3! \cdot \cancel{\frac{1}{2}}\, \cancel{\Gamma \frac{1}{2}} \times 2}{\frac{9}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \cancel{\frac{3}{2}} \cdot \cancel{\frac{1}{2}}\, \cancel{\Gamma \frac{1}{2}}} \right]$$

$$= 16\sqrt{2} \left[\frac{32}{315} \right]$$

$$\int_0^\infty \sqrt{x}\, e^{-x^2} dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\, dx = \frac{\pi}{2\sqrt{2}}$$

$$\sqrt{x} \cdot e^{-x^2} dx \qquad x^2 = t \quad x = \sqrt{t}$$

$$2x\, dx = dt$$

$$dx = \frac{dt}{2\sqrt{t}}$$

$$e^{-t} \cdot t^{1/4} \cdot \frac{dt}{2\sqrt{t}}$$

$$\frac{1}{2}\int_0^\infty e^{-t} \cdot t^{1/4 - 1/2}\, dt$$

$$\Rightarrow \frac{1}{2}\int_0^\infty e^{-t} \cdot t^{-1/4}\, dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t} \cdot t^{3/4 - 1}\, dt$$

$$= \frac{1}{2}\Gamma\frac{3}{4}.$$

$$\text{ii,} \rightarrow \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\, dx$$

$x^2 = t \quad x = \sqrt{t}$

$2x\,dx = dt$

$dx = \frac{dt}{2\sqrt{t}}$

$$\int_0^\infty \frac{e^{-t}}{t^{1/4}} \cdot \frac{dt}{2\sqrt{t}}$$

$$\Rightarrow \frac{1}{2}\int_0^\infty e^{-t} \cdot t^{-3/4}\, dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t}\, t^{1/4 - 1}\, dt = \frac{1}{2}\Gamma\frac{1}{4}$$

$$\rightarrow \frac{1}{2}\Gamma\frac{3}{4} \times \frac{1}{2}\Gamma\frac{1}{4} = \frac{1}{4}\Gamma\frac{3}{4}\,\Gamma\frac{1}{4}$$

$$= \frac{1}{4}\,\Gamma\frac{1}{4}\,\Gamma\left(1 - \frac{1}{4}\right) = \frac{1}{4} \cdot \frac{\pi}{\sin\frac{\pi}{4}}$$

$$= \frac{1}{4} \cdot \sqrt{2}\,\pi$$

$$= \frac{\pi}{2\sqrt{2}}$$

luate

$$\int_2^3 x^2y\,dx\,dy$$

$$\int_1^2\left[\int_2^3 x^2y\,dx\right]dy$$

$$\left(\frac{x^3}{3}\right)_2^3\left(\frac{y^2}{2}\right)_1^2$$

$$\left(\frac{27-8}{3}\right)\left(\frac{4-1}{2}\right)=$$

$$\frac{19}{\cancel{3}}\cdot\frac{\cancel{3}}{2}=\frac{19}{2}$$

$$\int_1^2\int_3^4 x e^{x+y}\,dx\,dy.$$

$$=\int_1^2\int_3^4 = \left[\int_1^2 e^y\,dy\right]\left[\int_3^4 x e^x\,dx\right]_4$$

$$=\left[e^y\right]_1^2\cdot\left[(xe^x)-\int 1\cdot e^x\,dx\right]_3$$

$$=\left(e^2-e^1\right)\left[(4e^4-3e^3)-(e^4-e^3)\right]$$

$= (e^2 - e^1)[3e^4 - 2e^3]$

$= 3e^6 - 3e^5 - 2e^5 + 2e^4$

$= 3e^6 - 5e^5 + 2e^4$

$$\int_1^2 \int_x^{x^2} x y^2 \, dx \, dy$$

$$= \frac{1}{3} \int_1^2 x \left(y^3\right)_x^{x^2} dx$$

$$= \frac{1}{3} \int_1^2 x \left(x^6 - x^3\right) dx$$

$$= \frac{1}{3} \int_1^2 \left(x^7 - x^4\right) dx$$

$$= \frac{1}{3} \left[\frac{x^8}{8} - \frac{x^5}{5}\right]_1^2 = \frac{1}{3}\left[\left(\frac{2^8}{8} - \frac{2^5}{5}\right) - \left(\frac{1}{8} - \frac{1}{5}\right)\right]$$

$$= \frac{1}{3}\left[\frac{\cancel{256}^{32}}{\cancel{8}} - \frac{32}{5} + \frac{3}{40}\right]$$

$$= \frac{1}{3}\left[\frac{128}{5} + \frac{3}{40}\right] = \frac{1}{3}\left[\frac{1024 + 3}{40}\right]$$

$$= \frac{1}{3}\left[\frac{1027}{40}\right].$$

$$= \frac{1027}{120}$$

$$\int \int_1^2 xy(1+x+y)\,dy\,dx$$

$$\int_1^2 \left[xy + x^2y + xy^2\,dy \right] dx.$$

$$\left[x\left(\frac{y^2}{2}\right)_1^2 + x^2\left(\frac{y^2}{2}\right)_1^2 + x\left(\frac{y^3}{3}\right)_1^2 \right]$$

$$\int_0^1 \left[\frac{x}{2}(3) + \frac{x^2}{2}(4-1) + \frac{x}{3}(7) \right]$$

$$\int_0^3 \left(\frac{3x}{2} + \frac{3x^2}{2} + \frac{7x}{3} \right)$$

$$= \frac{3}{4}(x^2)_0^3 + \frac{3}{6}(x^3)_0^3 + \frac{7}{6}(x^2)_0^3$$

$$= \frac{3}{4}(9) + \frac{\cancel{3}}{\cancel{6}_2}(27) + \frac{7}{\cancel{6}_2}(\cancel{9}^{3})$$

$$= \frac{27}{4} + \frac{27}{2} + \frac{21}{2} = \frac{27+54+42}{4}$$

$$= \frac{123}{4}.$$

$\rightarrow \int_0^2 \int_0^3 xy \, dx \, dy$

$\int_0^2 y \, dy \quad \int_0^3 x \, dx$

$\left[\frac{y^2}{2}\right]_0^2 \cdot \left[\frac{x^2}{2}\right]_0^3$

$\left(\frac{\cancel{4}^2}{\cancel{2}} \cancel{0}\right) \cdot \left(\frac{9}{\cancel{2}} \cancel{0} \right. \Rightarrow 9.$

$\rightarrow \int_0^2 \int_0^x e^{x+y} \, dy \, . \, dx.$

$\int_0^2 \int_0^x e^x \cdot e^y \, . \, dy \, . \, dx.$

$= \int_0^2 e^x \left[\int_0^x e^y \, dy \right] dx$

$\int_0^2 e^x \left(e^y\right)_0^x dx = \int_0^2 e^x \left(e^x - 1\right) dx.$

$= \int_0^2 e^{2x} - e^x \, dx.$

$= \left(\frac{e^{2x}}{2}\right)_0^2 - \left(e^x\right)_0^2 = \frac{e^4}{2} - e^2.$

$$f(s) = \log\left(\frac{s+3}{s+4}\right).$$

$$f(s) = \log(s+3) - \log(s+4)$$

$$f'(s) = \frac{1}{s+3} - \frac{1}{s+4}$$

$$L^{-1}\{f'(s)\} = L^{-1}\left\{\frac{1}{s+3}\right\} - L^{-1}\left\{\frac{1}{s+4}\right\}.$$

$$(-1)^1 t^1 \cdot f(t) = e^{-3t} - e^{-4t}$$

$$f(t) = \frac{e^{-3t} - e^{-4t}}{-t}$$

$$f(t) = \frac{e^{-4t} - e^{-3t}}{t}$$

$$L^{-1}\{f(s)\} = \frac{e^{-4t} - e^{-3t}}{t}$$

$$f(s) = \log\left(\frac{s+1}{s-1}\right)$$

$$f(s) = \log(s+1) - \log(s-1)$$

$$f'(s) = \frac{1}{s+1} - \frac{1}{s-1}.$$

$$L^{-1}\{f'(s)\} = L^{-1}\left\{\frac{1}{s+1}\right\} - L^{-1}\left\{\frac{1}{s-1}\right\}.$$

$$(-1)^1 t^1 \cdot f(t) = e^{-t} - e^{t}$$

$$f(t) = \frac{e^{t} - e^{-t}}{t}$$

$$L^{-1}\{f(s)\} = \frac{e^{t} - e^{-t}}{t}$$

→ find $L^{-1}\left\{\log\left(\frac{s^2+1}{(s-1)^2}\right)\right\}$.

$$f(s) = \log\left(\frac{s^2+1}{(s-1)^2}\right)$$

$$f(s) = \log(s^2+1) - \log(s-1)^2$$

$$f'(s) = \frac{1}{s^2+1}(2s) - \frac{2}{(s-1)}.$$

$$L^{-1}\{f'(s)\} = 2L^{-1}\left\{\frac{s}{s^2+1}\right\} - 2L^{-1}\left\{\frac{1}{s-1}\right\}.$$

$$(-1)^1 t^1 f(t) = 2\cos t - 2e^t$$

$$f(t) = \frac{2e^t - 2\cos t}{t}.$$

$$L^{-1}\{f(s)\} = \frac{2e^t - 2\cos t}{t}.$$

→ find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.

$$f(s) = \frac{s}{(s^2+a^2)^2}$$

$$f'(s) = \frac{(s^2+a^2)^2(1) - s\left[2(s^2+a^2)\cdot 2s\right]}{(s^2+a^2)^2}.$$

$$= \frac{s^2+a^2\left[s^2+a^2-4s^2\right]}{(s^2+a^2)^2}$$

$$= \frac{a^2-3s^2}{s^2+a^2}$$

$L^{-1}\left\{\frac{s}{s^2+a^2}\right\}$

$L^{-1}\{f(s)\} = f(t)$

$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \frac{1}{a}\sin at$

$L^{-1}\left\{\frac{d}{ds}\left(\frac{1}{s^2+a^2}\right)\right\} = (-1)^1 t^1 f(t).$

$L^{-1}\left\{\frac{-1\times 2s}{(s^2+a^2)^2}\right\} = (-1)\, t\, f(t).$

$L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{t}{2} f(t) = \frac{t}{2}\cdot\frac{1}{a}\times \sin at$

$L^{-1}\left\{\frac{1}{s(s+2)}\right\}.$

W.K.T

$L^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t} = f(t).$

By D by S.

$L^{-1}\left\{\frac{1}{s(s+2)}\right\} = \int_0^t f(x)\,dx = \int_0^t e^{-2x}\,dx = \frac{\left[e^{-2x}\right]_0^t}{-2} = \frac{\left[e^{-2t}-1\right]}{-2}$

$= \frac{1}{2} - \frac{e^{-2t}}{2}$

$\rightarrow L^{-1}\left\{\frac{1}{s^4-2s^3}\right\}$

$= L^{-1}\left\{\frac{1}{s^3(s-2)}\right\}$.

W.K.T $\quad L^{-1}\left\{\frac{1}{s-2}\right\} = e^{2t} = f(t)$.

$$L^{-1}\left\{\frac{1}{s^3(s-2)}\right\} = \int_0^t f(x)\,dx = \int_0^t e^{2x}\,dx = \frac{\left[e^{2x}\right]_0^t}{2} = \frac{e^{2t}-1}{2}$$

$$= \frac{e^{2t}}{2} - 1/2.$$

$$\int_0^t \left(\frac{e^{2x}}{2} - \frac{1}{2}\right)dx = \left[\frac{e^{2x}}{4} - \frac{x}{2}\right]_0^t = \left[\left(\frac{e^{2t}}{4} - \frac{1}{4}\right) - \left(\frac{t}{2}\right)\right]$$

$$= \left(\frac{e^{2t}}{4} - \frac{t}{2} - \frac{1}{4}\right)$$

$$\int_0^t \left(\frac{e^{2x}}{4} - \frac{x}{2} - \frac{1}{4}\right)dx = \left[\frac{e^{2x}}{8} - \frac{x^2}{4} - \frac{x}{4}\right]_0^t$$

$$= \left[\left(\frac{e^{2t}}{8} - \frac{t^2}{4} - \frac{t}{4}\right) - \left(\frac{1}{8}\right)\right]$$

$$= \left[\frac{e^{2t}}{8} - \frac{t^2}{4} - \frac{t}{4} - \frac{1}{8}\right]$$

olution Theorem:

If $L^{-1}\{f(s)\} = f(t)$

& $L^{-1}\{g(s)\} = g(t)$

en $f*g = L^{-1}\{f(s)\cdot g(s)\} = \int_0^t f(x)\,g(t-x)\,dx$.

id $L^{-1}\left\{\frac{1}{(s+2)(s-3)}\right\}$

$= L^{-1}\left\{\frac{1}{s+2}\cdot\frac{1}{s+3}\right\}$.

$L^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t} = f(t)$.

$L^{-1}\left\{\frac{1}{s-3}\right\} = e^{3t} = g(t)$.

volution Theorem

$L^{-1}\left\{\frac{1}{(s+2)(s-3)}\right\} = \int_0^t e^{-2x}\cdot e^{3(t-x)}\,dx$

$= \int_0^t e^{-2x}\cdot e^{3t}\cdot e^{-3x}\,dx.$

$= e^{3t}\int_0^t e^{-5x}\,dx$

$= \frac{e^{3t}}{-5}\left[e^{-5x}\right]_0^t = \frac{-e^{3t}}{5}\left[e^{-5t}-1\right]$

$= \underline{e^{3t}-e^{-2t}}$

By

$= \frac{1}{\cdot}$

$= -$

$=$

$=$

$\rightarrow \quad L^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$.

$$= L^{-1}\left\{\frac{1}{(s^2+2^2)}\cdot\frac{s}{(s^2+3^2)}\right\}$$

$$L^{-1}\left\{\frac{1}{(s^2+2^2)}\right\} = \frac{1}{2}\sin 2t = f(t)$$

$$L^{-1}\left\{\frac{s}{s^2+3^2}\right\} = \cos 3t = g(t).$$

By Convolution Theorem,

$$L^{-1}\left\{\frac{1}{(s^2+4)(s^2+9)}\right\} = \frac{1}{2}\int_0^t \sin 2x \cos 3(t-x)\,dx.$$

$$= \frac{1}{4}\int_0^t 2\sin 2x\cdot\cos(3t-3x)\,dx.$$

$$= \frac{1}{4}\int_0^t \left[\sin(2x+3t-3x)+\sin(2x-3t+3x)\right]dx.$$

$$= \frac{1}{4}\int_0^t \sin(3t-x)\,dx + \frac{1}{4}\int_0^t \sin(5x-3t)\,dx.$$

$$= -\frac{1}{4}\left[\frac{\cos(3t-x)}{-1}\right]_0^t - \frac{1}{4}\left[\frac{\cos(5x-3t)}{5}\right]_0^t$$

$$= \frac{1}{4}\left[\cos 2t - \cos 3t\right] - \frac{1}{20}\left[\cos 2t - \cos 3t\right].$$

$$= \frac{1}{5}\cos 2t - \frac{\cancel{1}}{5}\cos 3t$$

$$= \frac{\cos 2t - \cos 3t}{5}$$

$$L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}.$$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\cdot\frac{1}{s^2+a^2}\right\}$$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at = g(t)$$

$$L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin at = f(t).$$

Convolution Theorem.

$$L^{-1}\left\{\frac{1}{(s^2+a^2)(s^2+a^2)}\right\} = \frac{1}{a}\int_0^t \sin ax \cos a(t-x)\,dx.$$

$$= \frac{1}{2a}\int_0^t 2\sin ax \cdot \cos(at-ax)\,dx.$$

$$\frac{1}{2a}\int_0^t \left[\sin(ax+at-ax)+\sin(ax-at+ax)\right]dx.$$

$$\frac{1}{2a}\int_0^t \sin at \cdot dx + \frac{1}{2a}\int_0^t \sin(2ax-at)\cdot dx.$$

$$\frac{\sin at}{2a}[x]_0^t + \frac{-1}{2a}\left[\frac{\cos(2ax-at)}{2a}\right]_0^t$$

$$\frac{\sin at}{2a} - \frac{1}{4a^2}\left[\cos at - \cos at\right]$$

$$= \frac{t\sin at}{2a}$$

$$\therefore \quad L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{t\sin at}{2a}$$

$\rightarrow \quad L^{-1}\left\{\dfrac{1}{s(s+1)(s+2)}\right\}$

(i) $\quad L^{-1}\left\{\dfrac{1}{s(s+1)}\right\}$

W.K.T. $\quad L^{-1}\left\{\dfrac{1}{s}\right\} = 1 = f(t).$

$$L^{-1}\left\{\dfrac{1}{s+1}\right\} = e^{-t} = g(t)$$

By C. Theorem

$$L^{-1}\left\{\dfrac{1}{s(s+1)}\right\} = \int_0^t 1\cdot e^{-(t-x)}\,dx.$$

$$= e^{-t}\int_0^t e^{x}\,dx = e^{-t}\left[e^{x}\right]_0^t$$

$$= e^{-t}\left[e^{t}-1\right]$$

$$= 1-e^{-t} = f(t)$$

W.K.T.

$$L^{-1}\left\{\dfrac{1}{s+2}\right\} = e^{-2t} = g(t).$$

By C. Theorem.

$$L^{-1}\left\{\dfrac{1}{s(s+1)(s+2)}\right\} = \int_0^t \left[1-e^{-x}\right] e^{-2(t-x)}\,dx$$

$$\cancel{= \int_0^t \left[e^{-2(t-x)} - e^{-x}\, e^{-2(t-x)}\right] dx.}$$

$$\cancel{= \int_0^t \left(e^{-2t+2x}\right) = \left(e^{-2t}\cdot e^{2x}\right)}$$

$$e^{-2t}\int_0^t [e^{2x} - e^{x}]\,dx$$

$$e^{-2t}\left[\frac{(e^{2x})_0^t}{2} + (e^{+x})_0^t\right]$$

$$e^{-2t}\left[\frac{e^{2t}}{2} - \frac{1}{2} + e^{+t} - 1\right]$$

$$= \frac{1}{2} - \frac{e^{-2t}}{2} + e^{-t} - e^{-2t}$$

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}.$$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\cdot\frac{s}{s^2+b^2}\right\}.$$

$$\downarrow \qquad\qquad \downarrow$$

$$\cos at \qquad \cos bt$$

Theorem

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\} = \int_0^t \cos ax \cos b(t-x)\,dx$$

$$= \frac{1}{2}\int_0^t 2\cos ax \cos b(t-x)\,dx,$$

$$\frac{1}{2}\int_0^t \left[\cos(ax+bt-bx) + \cos(ax-bt+bx)\right]dx.$$

Applications of Laplace transformations to differential eqns:

① $L\{y'\} = sL\{y\} - y(0)$.

② $L\{y''\} = s^2 L\{y\} - sy(0) - y'(0)$.

③ $L\{y'''\} = s^3 L\{y\} - s^2 y(0) - sy'(0) - y''(0)$.

→ Using Laplace transform Method solve.

$(D^2+1)y = 6\cos 2t$, $t>0$ if $y=3$, $Dy=1$, when $t=0$

$y''+y = 6\cos 2t$, given $y(0)=3$, $y'(0)=1$

Taking L.T on B.S.

$L\{y'''\} + L\{y\} = 6L\{\cos 2t\}$.

$s^2 L\{y\} - sy(0) - y'(0) + L\{y\} = \dfrac{6\times s}{s^2+4}$.

$L\{y\}[s^2+1] - s(3) - 1 = \dfrac{6s}{s^2+4}$

$L\{y\}[s^2+1] = \dfrac{6s}{s^2+4} + 3s + 1$

$L\{y\} = \dfrac{6s}{(s^2+4)(s^2+1)} + \dfrac{3s+1}{s^2+1}$.

$y = L^{-1}\left\{\dfrac{6s}{(s^2+4)(s^2+1)}\right\} + L^{-1}\left\{\dfrac{3s+1}{s^2+1}\right\}$.

$= L^{-1}\left\{\dfrac{3s+1}{s^2+1^2}\right\}$

$$3L^{-1}\left\{\frac{s}{s^2+1}\right\} + L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$3\cos t + \sin t \longrightarrow (2)$$

$$\frac{}{)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1}$$

$$= (As+B)(s^2+1) + (Cs+D)(s^2+4)$$

$$s = As^3+Bs^2+As+B+Cs^3+Ds^2+4Cs+4D.$$

0	$B+D=0$	$A+4C=6$	$B+4D=0$
$-C$	$B=-D$	$-C+4C=6$	$-D+4D=0$
$=-2$	$B=0$	$3C=6$	$3D=0$
		$C=2$	$D=0$

$$\left.\frac{s}{{}^2+4)(s^2+1)}\right\} = -\frac{6}{3}L^{-1}\left\{\frac{s}{s^2+2^2}\right\} + \frac{6}{3}L^{-1}\left\{\frac{s}{s^2+1^2}\right\}.$$

$$= -2\cos 2t + 2\cos t \longrightarrow (3)$$

$$= 3\cos t + \sin t - 2\cos 2t + 2\cos t$$

$$= \sin t + 5\cos t - 2\cos 2t$$

Solve $\dfrac{d^2x}{dt^2} + \dfrac{3dx}{dt} + 2x = e^{-t}$, $x(0)=0, x'(0)=1$

$$x'' + 3x' + 2x = e^{-t}$$

$$L\{x''\} + 3L\{x'\} + 2L\{x\} = L\{e^{-t}\}$$

$$x\} - sx(0) - x'(0) + 3sL\{x\} - 3x(0) + 2L\{x\} = L\{e^{-t}\}$$

$$\}\left[s^2+3s+2\right] - 1 = \frac{1}{s+1}.$$

→ s

Giv

$4L$

$4s^2$

L

$$L\{x\}\left[s^2+3s+2\right] = \frac{1}{s+1}+1$$

$$L\{x\}\left[s^2+3s+2\right] = \frac{s+2}{s+1}$$

$$L\{x\}\left[(\cancel{s+2})(s+1)\right] = \frac{\cancel{s+2}}{s+1}$$

$$L\{x\} = \frac{1}{(s+1)^2}$$

$$x = L^{-1}\left\{\frac{1}{(s+1)^2}\right\}.$$

$$x = e^{-t}\cdot L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$x = e^{-t}\cdot t$$

$$x = te^{-t}$$

$\rightarrow$ solve $4y'' + \cancel{8}\pi^2 y = 0$.

Given $y(0) = 2,\ y'(0) = 0$.

$$4L\{y''\} + \pi^2 L\{y\} = 0$$

$$4s^2 L\{y\} - 4s\,y(0) - 4y'(0) + \pi^2 L\{y\} = 0.$$

$$L\{y\}\left[4s^2+\pi^2\right] - 8s = 0$$

$$L\{y\} = \frac{8s}{4s^2+\pi^2} = \frac{2s}{s^2+\frac{\pi^2}{4}}$$

$$y = L^{-1}\left\{\frac{2s}{s^2+\frac{\pi^2}{4}}\right\}.$$

$$y = 2\cos\frac{\pi}{2}t.$$

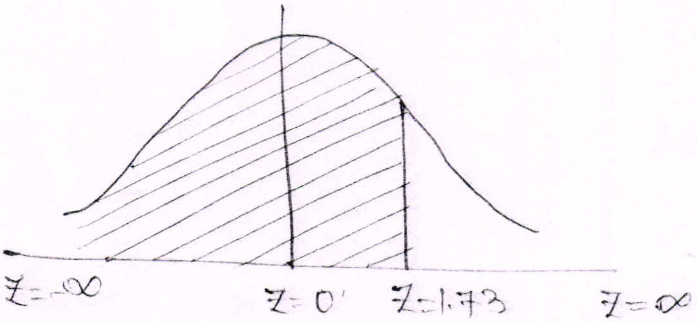
Find the area of standard Normal Curve which

(i) to the left of $Z = 1.73$.

to the Right of $Z = -0.66$

b/w $Z = -1.45$ and $Z = 1.45$

b/w $Z = 1.25$ and $Z = 1.67$

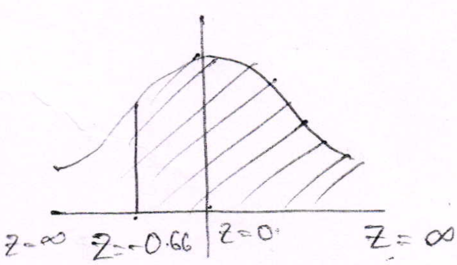


ea lying on the left side of $Z = 1.73$

$$= \int_{-\infty}^{0} f(z)\,dz + \int_{0}^{1.73} f(z)\,dz$$

$$= 0.5 + 0.4582$$ 1.73 → 1.7 under 3.

$$= 0.9582.$$



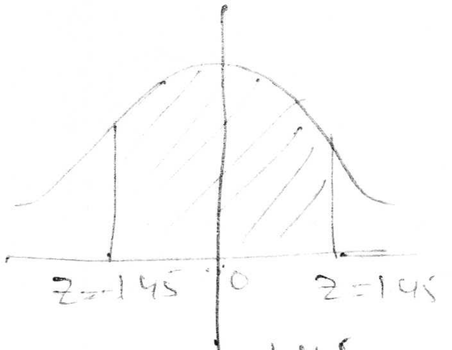
$$= \int_{-0.66}^{0} f(z)\,dz + \int_{0}^{\infty} f(z)\,dx.$$

$$= 0.2454 + 0.5$$

0.5000
0.2454

$$= 0.7454.$$

iii)

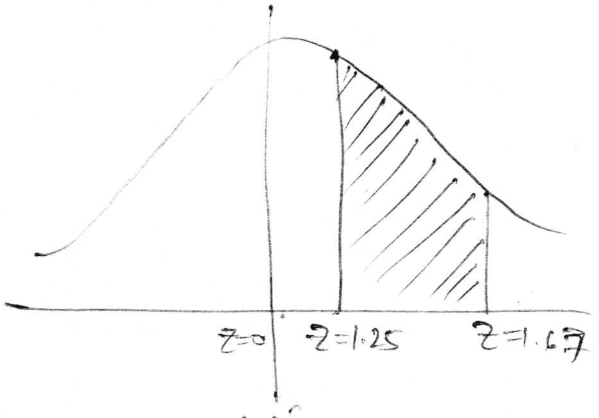


$$= \int_{-1.45}^{0} f(z)\,dz + \int_{0}^{1.45} f(z)\,dz$$

$$= 0.4265 + 0.4265$$

$$= 0.8530$$

iv) b/w $z = 1.25$ and $z = 1.67$



$$\Rightarrow \int_{0}^{1.67} f(z)\,dz - \int_{0}^{1.25} f(z)\,dz$$

$$= 0.4525 - 0.3944$$

$$= 0.0581$$

Weekly Wages of 1000 Workers are normally
tributed around a mean of Rupees 70 with as
) of Rs. 5. Estimate the no of Workers whose
ekly Wages will be

b/w Rs 68 and Rs 72.

More than Rs 80.

Less than Rs 63.

A Manufacturer Knows from the experience that
Resistance of Resistors he produces is Normal with
an 100Ω and S.D ~~20~~ Ω. What % of Resistors
ill have Resistance b/w 96Ω and 104Ω.

Let X Represents Weekly Wages of the Workers

$\mu = 70$, $\sigma = 5$, $N = 1000$

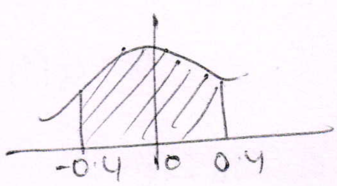
$P(68 \leq X \leq 72)$.

$$z = \frac{x-\mu}{\sigma}$$

At $x = 68 \Rightarrow z = \frac{68-70}{5} = -0.4$

$x = 72 \Rightarrow z = \frac{72-70}{5} = 0.4$

$P(-0.4 < z < 0.4)$.



$$\int_{-0.4}^{0} f(z)\,dz + \int_{0}^{0.4} f(z)\,dz$$

i, k

ii,

iii,

$= 0.1554 + 0.1554$

$= 0.3108$

i, Workers whose wages lies b/w 68 & 72 is 1000×0.3108

$= 310.8$

$\cong 311.$

ii, $P(X > 80)$.

$X = 80 \Rightarrow z = \dfrac{80-70}{5} = 2$

$P(z > 2)$

$$= \int_2^\infty f(z)\,dz$$

$$= \int_0^\infty f(z)\,dz - \int_0^2 f(z)\,dx$$

$$= 0.5 - 0.4772$$

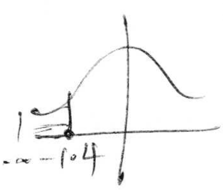
$$= 0.0228 \Rightarrow 1000 \times 0.0228$$

$$\Rightarrow 22.8$$

$\begin{array}{r} 0.5000 \\ 0.4772 \\ \hline 0.0228 \end{array}$

iii) $P(X < 63)$.

$X = 63 \Rightarrow z = \dfrac{63-70}{5} = -7/5 = -1.4$.



$$\Rightarrow \int_{-\infty}^{0} f(z)\,dx - \int_{-1.4}^{0} f(z)\,dz$$

$$= 0.5 - 0.4192$$

$$= 0.0808$$

$\begin{array}{r} 0.5000 \\ 0.4192 \\ \hline 0.0808 \end{array}$

$\Rightarrow 1000 \times 0.0808$

80.8

Let X Represents Resistance of Resistors

$\mu = 100$, $\sigma = 20$

$P(96 \leq X \leq 104) = ?$

$X = 96$, $z = \frac{96-100}{20} = -\frac{4}{20} = -0.2$.

$X = 104$, $z = \frac{104-100}{20} = \frac{4}{20} = +0.2$.

$P(-0.2 \leq X \leq 0.2)$.

$$= \int_{-0.2}^{0} f(z)\,dz + \int_{0}^{0.2} f(z)\,dz.$$

$= 0.0793 + 0.0793.$

$= 0.1586.$

→ solve the following differential eqn by laplace
transformation

$(D^2+4D+5)y = 5$

$y(0)=0, \quad y'(0)=0$

$y''+4y'+5y = 5$

$L\{y''\} + 4L\{y'\} + 5L\{y\} = L\{5\}.$

$s^2L\{y\} - sy(0) - y'(0) + 4(sL\{y\} - y(0)) + 5L\{y\} = 5/s$

$s^2L\{y\} + 4sL\{y\} + 5L\{y\} = 5/s.$

$L\{y\}(s^2+4s+5) = 5/s$

$$L\{y\} = \frac{5}{s^3+4s^2+5s}$$

$$= \frac{5}{s(s^2+4s+5)}.$$

$$y = 5L^{-1}\left\{\frac{1}{s(s^2+4s+5)}\right\},$$

$$L^{-1}\left\{\frac{1}{s^2+4s+5}\right\}$$

$$L^{-1}\left\{\frac{1}{(s+2)^2+1}\right\}.$$

$$\Rightarrow e^{-2t}\cdot L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$= e^{-2t}\cdot \sin t.$$

dividing by s

$$L^{-1}\left\{\frac{1}{s(s^2+4s+5)}\right\} = \int_0^t e^{-2x}\sin x\,dx$$

$$\cancel{\sin x\left(\frac{e^{-2x}}{-2}\right) - \int(\cos x)}$$

$$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$

$$y = 5L^{-1}\left\{\frac{1}{s(s^2+4s+5)}\right\}.$$

$$5\left(\frac{e^{-2x}}{5}\left[-2\sin x - \cos x\right]_0^t\right)$$

$$5\left(-\frac{e^{-2x}}{5}\left[2\sin x + \cos x\right]_0^t\right)$$

$$5\left[\frac{-2e^{-2x}\sin x}{5} - \frac{e^{-2x}\cos x}{5}\right]_0^t$$

$$5\left[\left(\frac{-2e^{-2t}\sin t}{5} - \frac{e^{-2t}\cos t}{5}\right) - \left(-\frac{1}{5}\right)\right]$$

$$5\left[\frac{1}{5} - \frac{2e^{-2t}\sin t}{5} - \frac{e^{-2t}\cos t}{5}\right]$$

$$= 1 - 2e^{-2t}\sin t - e^{-2t}\cos t.$$

→ solve $(D^2+9)y = \sin t$ Given that $y(0)=1,\ y(\pi/2)=1$

$$y''+9y = \sin t$$

$$L\{y''\} + 9L\{y\} = L\{\sin t\}.$$

$$s^2L\{y\} - sy(0) - y'(0) + 9L\{y\} = \frac{1}{s^2+1}$$

$$s^2L\{y\} - s - A + 9L\{y\} = \frac{1}{s^2+1}.$$

$$L\{y\}[s^2+9] - s - A = \frac{1}{s^2+1}.$$

$$L\{y\}[s^2+9] = \frac{1}{s^2+1} + s + A.$$

$$L\{y\} = \frac{1}{(s^2+1)(s^2+9)} + \frac{s}{s^2+9} + \frac{A}{s^2+9}.$$

$$\frac{1}{(s^2+1)(s^2+9)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9}$$

$$1 = (As+B)(s^2+9) + (Cs+D)(s^2+1)$$

$$1 = As^3 + Bs^2 + 9As + 9B + Cs^3 + Ds^2 + Cs + D.$$

$A+C=0$	$B+D=0$	$9A+C=0$	$9B+D=1.$
$A=-C$	$B=-D$	$-9C+C=0$	$-9D+D=1$
$A=0$	$B=1/8.$	$-8C=0$	$-8D=1$
		$C=0$	$D=-1/8.$

$$y = \frac{1}{8}L^{-1}\left\{\frac{1}{s^2+1}\right\} - \frac{1}{8}L^{-1}\left\{\frac{1}{s^2+9}\right\} + L^{-1}\left\{\frac{s}{s^2+9}\right\} + A\cdot L^{-1}\left\{\frac{1}{s^2+9}\right\}.$$

$$= \frac{1}{8}\cdot\sin t - \frac{1}{8}\sin 3t\cdot\frac{1}{3} + \cos 3t + \frac{A}{3}\sin 3t.$$

$$y = \frac{1}{8}\sin t - \frac{1}{24}\sin 3t + \cos 3t + \frac{A}{3}\sin 3t$$

$y\left(\frac{\pi}{2}\right) = 1$

$= \frac{1}{8} \sin \left(\frac{\pi}{2}\right) - \frac{1}{24} \sin 3\left(\frac{\pi}{2}\right) + \cos \frac{3\pi}{2} + \frac{A}{3} \sin 3\left(\frac{\pi}{2}\right)$

$\frac{1}{8} + \frac{1}{24} - \frac{A}{3}$

$= \frac{3 + 1 - 8A}{24}$

$24 = 4 - 8A$

$8A = -20$

$A = -20/8 = -5/2$

$= \frac{1}{8} \sin t - \frac{1}{24} \sin 3t + \cos 3t - \frac{5}{2} \sin 3t$

Solve by Laplace transform

$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$

Given $y(0) = 1$, $y'(0) = y''(0) = 2$.

$y''' + 2y'' - y' - 2y = 0$

$L\{y'''\} + 2L\{y''\} - L\{y'\} - 2L\{y\} = 0$

$L\{y\} - s^2 y(0) - s y'(0) - y''(0) + 2(s^2 L\{y\} - s y(0) - y'(0)) -$

$L\{y\} + y(0) - 2L\{y\} = 0$

$L\{y\} - s^2 - 2s - 2 + 2s^2 L\{y\} - 2s - 4 - s L\{y\} + 1 - 2L\{y\}$

$$s^3 L\{y\} + 2s^2 L\{y\} - s L\{y\} - 2 L\{y\} - 4s - s^2 - 5 = 0$$

$$L\{y\}\left[s^3 + 2s^2 - s - 2\right] = s^2 + 4s + 5$$

$$L\{y\} = \frac{s^2 + 4s + 5}{s^3 + 2s^2 - s - 2}$$

$$L\{y\} = \frac{s^2 + 4s + 5}{s^2(s+2) - 1(s+2)}$$

$$y = L^{-1}\left\{\frac{s^2 + 4s + 5}{(s+1)(s-1)(s+2)}\right\}.$$

$$\frac{s^2 + 4s + 5}{(s+1)(s-1)(s+2)} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+2}$$

$$s^2 + 4s + 5 = A(s-1)(s+2) + B(s+1)(s+2) + C(s+1)(s-1).$$

$$s^2 + 4s + 5 = A(s^2 + s - 2) + B(s^2 + 3s + 2) + C(s^2 - 1).$$

$$1 = A + B + C$$

$$1 = A + B + 7 - 4B$$

$$A - 3B = -6$$

$$A + 3B = 4$$

$$2A = -2$$

$$A = -1$$

$$A + 3B = 4$$

$$-1 + 3B = 4$$

$$3B = 5$$

$$B = 5/3$$

$$5 = \cancel{-2A} + 2B - C.$$

$$2 = \cancel{2A} + 2B + 2C$$

$$7 = 4B + C.$$

$$C = 7 - 4B.$$

$$C = 7 - 4\left(\frac{5}{3}\right)$$

$$= 1/3.$$

$$y = -1 \cdot L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{5}{3} L^{-1}\left\{\frac{1}{s-1}\right\} + \frac{1}{3} L^{-1}\left\{\frac{1}{s+2}\right\}.$$

$$= -1 \cdot e^{-t} + 5/3\, e^{t} + 1/3\, e^{-2t}$$

$$y = -e^{-t} + 5/3\, e^{t} + 1/3\, e^{-2t}$$

$(D^3 - D^2 + 4D - 4)y = 68 e^x \sin 2x$, $y = 1$, $Dy = -19$, $D^2y = -37$ at $x = 0$

$y''' - y'' + 4y' - 4y = 68 e^x \sin 2x$ $\quad y(0) = 1$

$L\{y'''\} - L\{y''\} + 4L\{y'\} - 4L\{y\} = 68\, L\{e^x \sin 2x\}$

$s^3L\{y\} - s^2 y(0) - s y'(0) - y''(0) - s^2 L\{y\} + s y(0) + y'(0)$

$+ 4(sL\{y\} - y(0)) - 4L\{y\} = 68\, L\{e^x \sin 2x\}$

$s^3L\{y\} - s^2(1) - s(-19) - (-37) - s^2L\{y\} + s(1) - 19 +$

$4(sL\{y\} - 1) - 4L\{y\} = 68\left(\frac{2}{(s-1)^2+4}\right)$

$L\{y\}(s^3 - s^2 + 4s - 4) - s^2 + 19s + 37 + s - 19 - 4 = 68\left(\frac{2}{(s-1)^2+4}\right)$

$L\{y\}(s^3 - s^2 + 4s - 4) - s^2 + 20s + 14 = 68\left(\frac{2}{(s-1)^2+4}\right)$

$$L\{y\} = \left[\frac{68(2)}{((s-1)^2+4)(s^3-s^2+4s-4)} + \frac{s^2-20s-14}{(s-1)(s^2+4)}\right]$$

$$\frac{s^2-20s-14}{(s-1)(s^2+4)} = \frac{A}{s-1} + \frac{Bs+C}{(s^2+4)}.$$

$s^2 - 20s - 14 = A(s^2+4) + (Bs+C)(s-1)$

$s^2 \Rightarrow \quad 1 = A + B$

$s \Rightarrow \quad -20 = -B + C$

Constant $\Rightarrow \quad -14 = -C$

$\boxed{C = 14}$

$B = C + 20$

$\boxed{B = 34}$

$y_p = \mathcal{L}^{-1}$ $A = 1 - B = 1 - 34$
$= -33$

$\therefore \mathcal{L}^{-1}\left\{\frac{s^2 - 20s - 14}{(s-1)(s^2+4)}\right\} = -33\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{34s+14}{s^2+4}\right\}$.

$= -33e^t + 34\cos 2t + \frac{14\sin 2t}{2} \longrightarrow (2)$

Consider

$$\frac{136}{(s-1)(s^2+4)(s^2-2s+5)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+4} + \frac{Ds+E}{s^2-2s+5}$$

$136 = A(s^4 - 2s^3 + 9s^2 - 8s + 20) + (Bs+C)(s^3 - 3s^2 + 7s - 5) + (Ds+E)(s^3 - 4s - s^2 - 4)$

$s^4 \Rightarrow A + B + D = 0$

$s^3 \Rightarrow -2A - 3B + C + E - D = 0$

$s^2 \Rightarrow 0 = 9A + 7B - 3C$.

$s \Rightarrow 0 = -8A - 5B + 7C - 4D + 4E$.

Const $\Rightarrow 136 = 20A - 5C - 4E$

UNIT 2 β & Γ FUNCTIONS

definite Integral $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is called β-function

denoted by $B(m,n)$

$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, $m>0$, $n>0$

function is also Known as Eulerian function of 1st Kind

Gamma Function:

The definite Integral $\int_0^\infty e^{-x}x^{n-1}\,dx$ where $n>0$ is Known as Gamma function and is denoted by Γn

$\Gamma n = \int_0^\infty e^{-x}\cdot x^{n-1}\,dx$, $n>0$.

Gamma function is also Known as Eulerian Integral of 2nd Kind

This integral doesnot Convert if $n \le 0$

Properties of β-Function:

Symmetry of β-Function

$B(m,n) = B(n,m)$.

$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

Put $1-x=y$

$-dx = dy$

$dx = -dy$

U.L $\Rightarrow 0 = y$

L.L $\Rightarrow 1 = y$

$\Rightarrow -\int_1^0 (1-y)^{m-1}\, y^{n-1}\,dy$

$\Rightarrow \int_0^1 y^{n-1}\cdot(1-y)^{m-1}\,dy$

$B(n,m)$

$\beta(m,n) = \beta(n,m)$ [illegible]

2. $\beta(m,n) = [illegible] \int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

LHS $= \beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

Put $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta\,d\theta$

U.L: $1 = \sin^2\theta$

$\theta = \pi/2$

L.L: $0 = \sin^2\theta$

$\theta = 0$

$\Rightarrow 2\int_0^{\pi/2} \sin^{2m-2}\theta \cdot \cos^{2n-2}\theta \cdot \sin\theta\cos\theta\,d\theta$

$= 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \cdot d\theta.$

3. $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

LHS $= \beta(m,n) = \int_0^1 x^{m-1} \cdot (1-x)^{n-1}\,dx.$

$\beta(m+1,n) = \int_0^1 x^{m} \cdot (1-x)^{n-1}\,dx.$

$\beta(m,n+1) = \int_0^1 x^{m-1} \cdot (1-x)^{n}\,dx$

$\beta(m+1,n) + \beta(m,n+1) = \int_0^1 x^{m-1}(1-x)^{n-1}\left[x + 1 - x\right]dx = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

$= \beta(m,n) = $ LHS

[illegible] of β-function :

To show $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

$$\int_0^\infty \frac{x^{m-1}\,dx}{(1+x)^{m+n}} = \int_0^\infty \frac{y^{n-1}\,dy}{(1+y)^{m+n}}$$

W.K.T

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Put $x = \frac{1}{1+y}$ $\Rightarrow$ U.L $\Rightarrow$ $1 = \frac{1}{1+y}$

$y = 0$

$dx = \frac{-1}{(1+y)^2}\,dy$ L.L $\to \infty$

$$\beta(m,n) = -\int_\infty^0 \frac{\left(\frac{1}{1+y}\right)^{m-1}}{\left(1-\frac{1}{1+y}\right)^{m+n}}\,dy \times \frac{+1}{(1+y)^2}$$

$$= +\int_0^\infty \frac{\left(\frac{1}{1+y}\right)^{m} \times 1+y}{\left(\frac{y+2}{y+1}\right)^{m+n}} \frac{1}{(1+y)^2}\,dy$$

$$= \int_0^\infty \frac{1}{(1+y)^{m-1}} \times \frac{(1+y)^{m+n}}{(2+y)^{m+n}} \times \frac{1}{(1+y)^2}\,dy$$

$$= \int_0^\infty \frac{1}{(1+y)^{m-1}} \times \left(\frac{1+y-1}{1+y}\right)^{n-1} \times \frac{1}{(1+y)^2}\,dy$$

$$= \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$$ [By symmetry ($\because B(m,n) = B(n,m)$)]

Put $y = x \Rightarrow dy = dx$.

$$= \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx.$$

→ Show that $\int_0^{\pi/2} \sin^m\theta \cdot \cos^n\theta \, d\theta = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

~~Sol~~

W.K.T. $2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta = B(m,n)$

$$\frac{1}{2} B(m,n) = \int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$$

Put $2m-1 = p$, $2n-1 = q$

$$\frac{1}{2} B(m,n) = \int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \, d\theta$$

$m = \frac{p+1}{2}$, $n = \frac{q+1}{2}$

$$\frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right) = \int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \, d\theta$$ put $p = m, q = n$.

$$\frac{1}{2} B\left(\frac{m+1}{2}, \frac{n+1}{2}\right) = \int_0^{\pi/2} \sin^m\theta \cdot \cos^n\theta \, d\theta.$$

Express the following function in terms of B-function

$$\int_0^1 \frac{x \cdot dx}{\sqrt{1-x^2}}$$

Put $x^2 = y$

$$2x = \cancel{y} \cdot \frac{dy}{dx}.$$

$$x\,dx = \frac{dy \cdot \cancel{y}}{2}$$

$$\int_0^1 \frac{dy/2}{\sqrt{1-y}} = \frac{1}{2}\int_0^1 (1-y)^{-1/2} dy.$$

$$= \frac{1}{2}\int_0^1 y^{1-1} \cdot (1-y)^{1/2-1} dy = \frac{1}{2} B(1, 1/2).$$

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx.$$

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

$$x^2 = 9y$$

$$2x\,dx = 9\,dy$$

$$x\,dx = \frac{9\,dy}{2}$$

$$\int_0^{\cancel{3}} \frac{9\,dy}{2 \cdot 3\sqrt{y} \cdot \sqrt{9-9y}}$$

$$= \int_0^1 \frac{9\,dy}{2 \cdot 3 \cdot 3 \cdot \sqrt{y}\sqrt{1-y}} \Rightarrow \frac{1}{2}\int_0^1 \frac{dy}{\sqrt{y}\sqrt{1-y}}$$

$$= \frac{1}{2}\int_0^1 y^{-1/2}(1-y)^{-1/2} dy.$$

$$\Rightarrow \frac{1}{2}\int_0^1 y^{1/2-1}\cdot(1-y)^{1/2-1}\,dy$$

$$= \frac{1}{2} B\left(\frac{1}{2}, \frac{1}{2}\right)$$

$\rightarrow$ S.T

$$\int_{-1}^{1} (1+x)^{m-1}(1-x)^{n-1}\,dx = 2^{m+n-1} B(m,n).$$

W.K.T

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx \longrightarrow (1)$$

Let $x = \frac{1+y}{2}$ in (1)

$$dx = \frac{dy}{2}$$

$U.L = y = 1$

$L.L \Rightarrow y = -1$

$$\Rightarrow \int_{-1}^{1} \left(\frac{1+y}{2}\right)^{m-1}\left(1-\left(\frac{1+y}{2}\right)\right)^{n-1} dx.$$

$$= \int_{-1}^{1} \frac{(1+y)^{m-1}}{2^{m-1}} \frac{(1-y)^{n-1}}{2^{n-1}} \cdot \frac{dy}{2}.$$

$$= \int_{-1}^{+1} \frac{(1+y)^{m-1}(1-y)^{n-1}\,dy}{2^{m-1+1+n-1}}$$

$$B(m,n) = \int_{-1}^{+1} \frac{(1+x)^{m-1}(1-x)^{n-1}}{2^{m+n-1}}\,dx.$$ Put $y = -x \Rightarrow dy = -dx$

$$= \frac{1}{2^{m+n-1}} \int_{-1}^{+1} (1+x)^{m-1} \cdot (1-x)^{n-1}\,dx$$

$$2^{m+n-1} \cdot B(m,n) = \int_{-1}^{+1} (1+x)^{m-1}(1-x)^{n-1}\,dx.$$

S.T $$\int_0^{\infty} \frac{x^{m-1}\,dx}{(x+a)^{m+n}} = a^{-n} B(m,n).$$

W.K.T.

$$B(m,n) = \int_0^{\infty} \frac{x^{m-1}\,dx}{(x+1)^{m+n}}$$

$$x = \frac{t}{a} \qquad dx = \frac{dt}{a}$$

$$\int_0^{\infty} \frac{(t/a)^{m-1}}{\left(\frac{t}{a}+1\right)^{m+n}} \cdot \frac{dt}{a}.$$

$$\int_0^{\infty} \frac{t^{m-1} \cdot a^{-m+1}}{(a+t)^{m+n} \cdot a^{-m-n}} \cdot \frac{dt}{a}$$

$$= \int_0^{\infty} \frac{t^{m-1} \cdot a^{-m+1}}{(a+t)^{m+n}\; a^{-m-n+1}} \cdot dt$$

$$\Rightarrow \int_0^\infty \frac{t^{m-1}}{(t+a)^{m+n}} a^{-m+1+m+n-1} dt$$

$$B(m,n) \Rightarrow \int_0^\infty \frac{t^{m-1}}{(t+a)^{m+n}} a^n . dt$$

$$\Rightarrow \int_0^\infty \frac{t^{m-1}}{(t+a)^{m+n}} dt = a^{-n} B(m,n)$$

$$t = x, \; dt = dx$$

$$\Rightarrow \int_0^\infty \frac{x^{m-1}}{(x+a)^{m+n}} dx = a^{-n} B(m,n)$$

$$\rightarrow \int_0^1 \frac{x^{m-1}(1-x)^{n-1} dx}{(x+a)^{m+n}} = \frac{B(m,n)}{a^n(1+a)^m}.$$

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx. \longrightarrow (1)$$

$$x = \frac{t(a+1)}{t+a} \text{ in } (1) \Rightarrow \quad U.L = 1 = \frac{at+t}{t+a} \Rightarrow t = 1$$

$$L.L = 0 = \frac{t(a+1)}{t+a} \Rightarrow 0 = t(a+1), \; t = 0$$

$$dx = \left[\frac{(t+a)(a+1) - t(a+1).1}{(t+a)^2}\right] dt$$

$$dx = \frac{a[a+1]}{(t+a)^2} dt$$

$$(m,n) = \int_0^1 \left[\frac{t(a+1)}{t+a}\right]^{m-1} \left[1 - \frac{t(a+1)}{t+a}\right]^{n-1} \times \frac{a(a+1)}{(t+a)^2}\, dt$$

$$= \int_0^1 \frac{t^{m-1}(1+a)^{m-1}}{(t+a)^{m-1}} \times \left[\frac{t+a-at-t}{t+a}\right]^{n-1} \times \frac{a(a+1)}{(t+a)^2}\, dt$$

$$= a^{n-1} \int_0^1 \frac{t^{m-1}(1+a)^{m-\cancel{1}+\cancel{1}}(1-t)^{n-1}}{(t+a)^{m-\cancel{1}+n-\cancel{1}+\cancel{2}}}\, dt$$

$$B(m,n) = a^n(1+a)^m \int_0^1 \frac{t^{m-1}(1-t)^{n-1}}{(t+a)^{m+n}}\, dt$$

Put $t = x$

$dt = dx$.

$$\frac{B(m,n)}{(1+a)^m} = \int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}}\, dx.$$

ST $B(m,n) = \int_0^1 \frac{[x^{m-1} + x^{n-1}]\, dx}{(1+x)^{m+n}}$.

i-T from the I^{st} form of β-function

$$B(m,n) = \int_0^\infty \frac{x^{m-1}\, dx}{(1+x)^{m+n}}.$$

$$= \int_0^1 \frac{x^{m-1}\, dx}{(1+x)^{m+n}} + \int_1^\infty \frac{x^{m-1}\, dx}{(1+x)^{m+n}} \longrightarrow (2)$$

$\Rightarrow$ Put $x = 1/y$ in $\displaystyle\int_1^\infty \frac{x^{m-1}\,dx}{(1+x)^{m+n}}$

$$= \int_1^0 \frac{(1/y)^{m-1} \times (-1/y^2)\,dy}{(1+1/y)^{m+n}}.$$

$$\Rightarrow + \int_0^1 \frac{\frac{1}{y^{m-1+2}}\,dy}{\left(\frac{y+1}{y}\right)^{m+n}}$$

$$+ \int_0^1 \frac{\frac{1}{y^{m+1}}}{\left(\frac{y+1}{y}\right)^{m+n}}\,dy$$

$$= \int_0^1 \frac{1}{y^{m+1}} \times \frac{y^{m+n}}{(y+1)^{m+n}}\,dy.$$

$$= \int_0^1 \frac{y^{\cancel{m}+n-\cancel{m}-1}}{(y+1)^{m+n}}\,dy$$

$$\int_0^1 \frac{y^{n-1}}{(1+y)^{m+n}}\,dy.$$

Put $y = x \Rightarrow dy = dx$.

$$\Rightarrow \int_0^1 \frac{x^{n-1}\,dx}{(1+x)^{m+n}} \longrightarrow (3)$$

Put in (2)

$$B(m,n) = \int_0^1 \frac{\left[x^{m-1} + x^{n-1}\right]dx}{(1+x)^{m+n}}$$

3 | January | 2017

UNIT - 1 LAPLACE TRANSFORMA

Let $f(t,s)$ is a function of two variab t and s. The laplace transformation of the function $f(t)$ is denoted by $L\{f(t)\}$ and is defined as

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt \quad \forall\ t \geq 0$$

This is known as right sided laplace transformation or unilateral laplace transformation.

The laplace transformation of the function $f(t)$ exist if the integral is convergent $\rightarrow$ (other than $\pm\infty$)

* $e^{-\infty} = 0$
* $e^{\infty} = \infty$
* $e^{0} = 1$

1. $$L\{1\} = \int_0^{\infty} e^{-st}\,dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= \left[\frac{e^{-\infty} - e^{0}}{-s}\right]$$

$$= \frac{0-1}{-s} = \frac{-1}{-s} = \frac{1}{s} \quad \text{where } s>0.$$

2. $$L\{e^{at}\} = \int_0^{\infty} e^{-st}\cdot e^{at}\,dt$$

$$= \int_0^{\infty} e^{-t(s-a)}\,dt$$

$$= \frac{\left(e^{-t(s-a)}\right)_0^{\infty}}{-(s-a)} = \frac{e^{-\infty} - e^{0}}{-(s-a)} = \frac{1}{s-a}, \quad s>a$$

3. IIIly $L\{e^{-at}\} = \frac{1}{s+a}$, $s>-a$.

4. $$L\{\sinh at\} = \int_0^{\infty} e^{-st}\cdot \sinh at\,dt = \int_0^{\infty} e^{-st}\left[\frac{e^{at} - e^{-at}}{\ }\right]$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-t(s-a)}\,dt - \int_0^\infty e^{-t(s+a)}\,dt\right]$$

$$= \frac{1}{2}\left[\frac{\left(e^{-t(s-a)}\right)_0^\infty}{-(s-a)} - \frac{\left(e^{-t(s+a)}\right)_0^\infty}{-(s+a)}\right]$$

$$= \frac{1}{2}\left[\frac{0+1}{+(s-a)} - \frac{(0+1)}{+(s+a)}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{s}+a-\cancel{s}-(-a)}{s^2-a^2}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}a}{s^2-a^2}\right] = \frac{a}{s^2-a^2}$$

4. $L\{\cosh at\} = \int_0^\infty e^{-st}.\cosh at.$

Sol :- $\int_0^\infty e^{-st}\left[\frac{e^{+at}+e^{-at}}{2}\right]dt = \frac{1}{2}\left[\int_0^\infty e^{-t(s-a)}\,dt + e^{-t(s+a)}\,dt\right]$

$$= \frac{1}{2}\left[\frac{\left(e^{-t(s-a)}\right)_0^\infty}{-(s-a)} + \frac{\left(e^{-t(s+a)}\right)_0^\infty}{-(s+a)}\right]$$

$$= \frac{1}{2}\left[\frac{0+1}{+(s-a)} + \frac{0+1}{+(s+a)}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s-a)} + \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{s+\cancel{a}+s-\cancel{a}}{s^2-a^2}\right] = \frac{s}{s^2-a^2}$$

5. $L\{\sinh at\} = \frac{a}{s^2-a^2}$

6. $L\{t\} = \int_0^\infty e^{-st}\,t\,.dt.$

$\int e^{ax}.x\,dx$

$= \frac{e^{ax}}{a}\left(x-\frac{1}{a}\right)$

$$= \left[\frac{e^{-st}}{-s}\left(t+\frac{1}{ts}\right)\right]_0^\infty$$

$$= 0 + \left[\frac{1}{ts}\left(\frac{1}{s}\right)\right] = 1/s^2$$

7. $L\{\sin at\} = \int_0^\infty e^{-st} \cdot \sin at \, dt.$

sol:- * $\int e^{ax} \cdot \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b\cos bx\right]$

* $\int e^{ax} \cdot \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$

sol:- $L\{\sin at\} = \int_0^\infty e^{-st} \cdot \sin at \, dt$

$$= \left[\frac{e^{-st}}{s^2+a^2}\left[-s\sin at - a\cos at\right]\right]_0^\infty$$

$(s+a)$
dt

$$= 0(\ ) - \left[\frac{1}{s^2+a^2}\left[-s(0) - a(1)\right]\right]$$

$$= \frac{a}{s^2+a^2} \quad \text{where } a>0.$$

8. $L\{\cos at\} = \int_0^\infty e^{-st} \cdot \cos at \, dt$

sol:- $\left[\frac{e^{-st}}{s^2+a^2}\left[-s\cos at + a\sin at\right]\right]_0^\infty$

$$= \left[0(0) - \left[\frac{1}{s^2+a^2}(-s(1)+0)\right]\right]$$

$$= \frac{s}{s^2+a^2}$$

$\therefore L\{\cos at\} = \frac{s}{s^2+a^2}, \; s>0.$

S.NO	$L\{f(t)\}$	$f(s)$	
1.	$L\{1\}$	$\frac{1}{s}$, $s>0$	$\Gamma 1 = 1$
2	$L\{t\}$	$\frac{1}{s^2}$, $s>0$	$\Gamma \frac{1}{2} = \sqrt{\pi}$
3.	$L\{e^{at}\}$	$\frac{1}{s-a}$, $s>a$	$\Gamma n = (n-1)(n-2) \ldots \times (n-r)\Gamma n-r$
4.	$L\{e^{-at}\}$	$\frac{1}{s+a}$, $s>-a$	where $(n-r)>0$
5.	$L\{\sin at\}$	$\frac{a}{s^2+a^2}$, $s>0$	if n is +ve rational number.
6.	$L\{\sinh at\}$	$\frac{a}{s^2-a^2}$, $s>0$	
7.	$L\{\cos at\}$	$\frac{s}{s^2+a^2}$, $s>0$	
8.	$L\{\cosh at\}$	$\frac{s}{s^2-a^2}$, $s>0$	
9.	$L\{t^n\}$ $n \in z^+$	$\frac{n!}{s^{n+1}}$, $s>0$	
10.	$L\{t^a\}$ $a \notin z^+$	$\frac{\Gamma a+1}{s^{a+1}}$, $s>0$	
11.	$\Gamma n = \frac{\Gamma n+1}{n}$	if a is -ve rational number.		

$$1 = s(A+B) + Ab + aB$$

$\therefore$ On comparing both sides $A+B=0$

$\therefore A = -B$

$\Rightarrow Ab + aB = 1$

$\Rightarrow Ab - aA = 1$

$A(b-a) = 1 \quad \therefore A = \frac{1}{b-a}$

$\therefore B = \frac{+1}{a-b}$

$$\Rightarrow (b-a) \times \int_s^\infty \left(\frac{1}{(b-a)(s+a)} + \frac{1}{(a-b)(s+b)} \right) ds$$

$$\Rightarrow \frac{\cancel{(b-a)}}{\cancel{(b-a)}} \times \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b} \right) ds$$

$$\Rightarrow \left(\log(s+a) - \log(s+b) \right)_s^\infty$$

$$\Rightarrow \left[\log \frac{(s+a)}{(s+b)} \right]_s^\infty$$

$$\Rightarrow \lim_{s\to\infty} \log\left(\frac{s+a}{s+b}\right) - \log\left(\frac{s+a}{s+b}\right)$$

$$\Rightarrow \lim_{s\to\infty} \log\left(\frac{s(1+\frac{a}{s})}{s(1+\frac{b}{s})}\right) + \log\left(\frac{s+b}{s+a}\right)$$

$$\Rightarrow \lim_{s\to\infty} \log\left(\frac{\cancel{s}}{\cancel{s}}\right) + \log\left(\frac{s+b}{s+a}\right)$$

$$\Rightarrow \log\left(\frac{s+b}{s+a}\right)$$

3. $L\left\{\frac{\cos t}{t}\right\}$

Sol:- $f(t) = \cos t \quad \therefore f(s) = \frac{s}{s^2+1}$

$$L\left\{\frac{\cos t}{t}\right\} = \frac{1}{2}\int_s^\infty \frac{2s}{s^2+1}\, ds = \frac{1}{2}\left[\log(s^2+1)\right]^\infty$$

Division by t: If $L\{f(t)\} = f(s)$

if $L\{f(t)\} = f(s)$ then (a) $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s)\,ds$

(b) $L\left\{\frac{f(t)}{t^n}\right\} = \int_s^\infty f(s)\,(ds)^n$

Find 1. $L\left\{\frac{\sin at}{t}\right\}$

sol:-

$f(t) = \sin at \quad f(s) = \frac{a}{s^2+a^2}$

$L\left\{\frac{\sin at}{t}\right\} = \int_s^\infty \frac{a}{s^2+a^2}\cdot ds$

$= \frac{\cancel{a}}{\cancel{a}}\left[\tan^{-1}\left(\frac{s}{a}\right)\right]_s^\infty$

$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{a}\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{a}\right)$

$= \cot^{-1}(s/a)$.

2. $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$

sol:- $f(t) = e^{-at} - e^{-bt}$ then $L\{f(t)\} = L\{e^{-at} - e^{-bt}\}$

$= \frac{1}{s+a} - \frac{1}{s+b}$

$= \frac{\cancel{s}+b-\cancel{s}-a}{(s+a)(s+b)} \Rightarrow \frac{b-a}{(s+a)(s+b)} = f(s)$

then $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\} = \int_s^\infty \frac{b-a}{(s+a)(s+b)}\,ds$

$= (b-a)\int_s^\infty \frac{1}{(s+a)(s+b)}\,ds$

from the above equation $\frac{1}{(s+a)(s+b)} = \frac{A}{(s+a)} + \frac{B}{(s+b)}$

(1) $L\{t^2 \sin at\}$

Sol: $f(t) = \sin at \rightarrow f(s) = \dfrac{a}{s^2+a^2}$

Multiplying with t^2 then $L\{t^2 \sin at\}$

$$= (-1)^2 \frac{d^2}{ds^2}\left[\frac{a}{s^2+a^2}\right]$$

$$= a\frac{d}{ds}\left[\frac{-1(2s)}{(s^2+a^2)^2}\right]$$

$$= -2a.\frac{d}{ds}\left[\frac{s}{(s^2+a^2)^2}\right]$$

$$= -2a\left[\frac{(s^2+a^2)^2(1) - s2(s^2+a^2)(2s)}{(s^2+a^2)^{4}}\right]$$

$$= -2a\left[\frac{\cancel{(s^2+a^2)}\left(s^2+a^2 - 4s^2\right)}{(s^2+a^2)^{3}}\right]$$

$$= \frac{-2a\left[a^2-3s^2\right]}{(s^2+a^2)^3}.$$

2. $L\{t^3 . e^{3t}\}$. $f(t) = e^{3t}$ then $f(s) = \dfrac{1}{s-3}$

$$L\{t^3.e^{3t}\} = (-1)^3 . \frac{d^3}{ds^3}\left[\frac{1}{s-3}\right]$$

$$= + \frac{d^2}{ds^2}\left[\frac{+1}{(s-3)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{-2}{(s-3)^3}\right]$$

$$L\{t^3.e^{3t}\} = \frac{6}{(s-3)^4}.$$

6/January/2017

Multiplication by t^n

If $L\{f(t)\} = f(s)$ then

(a). $L\{t \cdot f(t)\} = -\frac{d}{ds}[f(s)]$

(b) $L\{t^n \cdot f(t)\} = (-1)^n \frac{d^n}{ds^n}(f(s))$

$t > a$

$t < a$

1. find $L\{t^2 \cdot \sin at\}$

Sol:- $f(t) = \sin at$ then $f(s) = \frac{a}{s^2+a^2}$

by Multiplication by t^2.

$L\{t^2 \cdot (\sin at)\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left(\frac{a}{s^2+a^2}\right)$

$= 1 \cdot \frac{d}{ds}\left[\frac{1 \cdot \cancel{a}}{\cancel{a}} \operatorname{Tan}^{-1}\left(\frac{s}{a}\right)\right]$

$= \frac{1}{\cancel{a}} \cdot \frac{a^2}{s^2+a^2} = \frac{a}{s^2+a^2}$ ✗

2. $L\{t(\cos t + \sin t)\}$

Sol:- $f(t) = (\cos t + \sin t)$ then $f(s) = \frac{s}{s^2+a^2} + \frac{1}{s^2+a^2}$

$f(s) = \frac{s+1}{s^2+1}$

multiplication by t

$L\{t(\cos t + \sin t)\} = -\frac{d}{ds}\left(\frac{s+1}{s^2+1}\right)$

$= (-1)\left[\frac{(s^2+1)(1) - (s+1)(2s)}{(s^2+1)^2}\right]$

$= (-1)\frac{\left[s^2+1 - 2s^2 - 2s\right]}{(s^2+1)^2}$

$$L\{f(\tfrac{t}{2})\} = 2\cdot f(2s)$$

$$= 2\left[\frac{(2s)^2 + 2\cdot(2s) + 2}{(2s-3)(2s+4)}\right]$$

$$L\{f(\tfrac{t}{2})\} = 2\left[\frac{4s^2 + 4s + 2}{(2s-3)(2s+4)}\right]$$

SECOND SHIFTING THEOREM :-

If $L\{f(t)\} = f(s)$ and $g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t < a \end{cases}$

then $L\{g(t)\} = e^{-as}\cdot f(s)$

1. Find $L\{g(t)\}$ where $g(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a. \end{cases}$

Sol:- We know that $L\{f(t)\} = L\{e^{1t}\}$

$$= \frac{1}{s-1} = f(s)$$

By second shifting theorem:

$$L\{g(t)\} = e^{-as} \times f(s)$$

$$= e^{-as} \times \frac{1}{s-1}$$

2. $g(t) = \begin{cases} \sin(t-\pi), & t > \pi \\ 0, & t < \pi \end{cases}$

Sol:- $e^{-\pi s} \times \frac{1}{s^2+1} = L\{g(t)\}$

3. $g(t) = \begin{cases} \cos(t - \pi/3), & t > \pi/3 \\ 0, & t < \pi/3 \end{cases}$

$$L\{g(t)\} = e^{-\pi/3\cdot s}\left(\frac{s}{1+s^2}\right)$$

CHANGE OF SCALE PROPERTY:

If $L\{f(t)\} = f(s)$ then 1. $L\{f(at)\} = \frac{1}{a} f(s/a)$

2. $L\{f(\frac{t}{a})\} = a \cdot f(as)$

Proof: we know that $L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = f(s)$

$$L\{f(at)\} = \int_0^\infty e^{-st} \cdot f(at)\,dt$$

put $at = x$

$a \cdot dt = dx$.

$$\Rightarrow \frac{1}{a}\int_0^\infty e^{-s \cdot \frac{x}{a}} \cdot f(x) \cdot dx$$

$$\Rightarrow \frac{1}{a}\int_0^\infty e^{-x \cdot (\frac{s}{a})} \cdot f(x) \cdot dx.$$

put $x = t$

$dx = dt$ $\Rightarrow \frac{1}{a}\int_0^\infty e^{-t \cdot (\frac{s}{a})} \cdot f(t) \cdot dt$

$$\Rightarrow \frac{1}{a} f(s/a) = L\{f(at)\}.$$

-a)

111'y $L\{f(\frac{t}{a})\} = a \cdot f(as)$

1. If $L\{f(t)\} = \dfrac{s^2+2s+2}{(s-3)(s+4)}$ find $L\{f(3t)\}$ (i)

(ii) $L\{f(\frac{t}{3})\}$

Sol: given $L\{f(t)\} = \dfrac{s^2+2s+2}{(s-3)(s+4)}$

s +5)

By change of scale properties

$$L\{f(3t)\} = \frac{1}{3} f(s/3)$$

$$= \frac{1}{3}\left[\frac{\frac{s^2}{9} + \frac{2s}{3} + 2}{(\frac{s}{3}-3)(\frac{s}{3}+4)}\right]$$

$$= \frac{1}{3}\left[\frac{(s^2+6s+18) \times \cancel{3} \times \cancel{3}}{\quad}\right] = \frac{1}{3}\left(\frac{s^2+6s+18}{\quad}\right)$$

By first shifting theorem

$$L\{e^{-t}\cdot(\sin t+\cos t)\} = \frac{1+s+1}{(s+1)^2+1} = \frac{s+2}{s^2+2s+2}$$

3. $L\{e^{2t}\cdot\cos 2t\}$

Sol :- $L\{\cos 2t\} = \frac{s}{s^2+4} = f(s)$

By the theorem $L\{e^{2t}\cdot\cos 2t\} = \frac{s-2}{(s-2)^2+4}$

$$= \frac{s-2}{s^2-4s+8}$$

4. Find $L\{e^{2t}\cdot\sin 3t\cdot\cos 2t\}$

Sol :- $L\{f(t)\} = \frac{1}{2}L\{2\sin 3t\cdot\cos 2t\}$

$$= \frac{1}{2}L\{\sin 5t+\sin t\}$$

$$= \frac{1}{2}\left[\frac{5}{s^2+25}+\frac{1}{s^2+1}\right] = f(s)$$

$\therefore$ From theorem $L\{e^{2t}\sin 3t\cdot\cos 2t\} = f(s-a)$

$$= \frac{1}{2}\left[\frac{5}{(s-2)^2+25}+\frac{1}{(s-2)^2+1}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{6(s-2)^2+30}{((s-2)^2+25)((s-2)^2+1)}\right]$$

$$\Rightarrow \frac{3(s-2)^2+15}{(s^2-4s+29)(s^2-4s+5)} = \frac{3s^2-12s+27}{(s^2-4s+29)(s^2-4s+5)}$$

$$\therefore L\{e^{2t}\cdot\sin 3t\cdot\cos 2t\} = \frac{3s^2-12s+27}{(s^2-4s+29)(s^2-4s+5)}$$

5/January/2016.

FIRST SHIFTING THEOREM (or) (Translation theorem)

If $L\{f(t)\} = f(s)$

then (a). $L\{e^{at}.f(t)\} = f(s-a)$

(b) $L\{e^{-at}.f(t)\} = f(s+a)$.

1. $f(s) = L\{f(t)\} = \int_0^\infty e^{-st}.f(t)\,dt$

$$f(s-a) = \int_0^\infty e^{-(s-a)t}.f(t)\,dt$$

$$= \int_0^\infty e^{-st}.e^{at}.f(t)\,dt$$

$$f(s-a) = L\{e^{at}f(t)\}$$

1. Find $L\{e^{2t}.\sin^2 t\}$

Sol :- $L\{\sin^2 t\} = \frac{1}{2}L\{1-\cos 2t\}$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{s^2}+\cancel{4}^{2}-\cancel{s^2}}{s(s^2+4)}\right] = \frac{2}{s(s^2+4)}.$$

∴ By first shifting theorem $f(s)$

$$L\{e^{2t}.\sin^2 t\} = \frac{2}{(s-2)((s-2)^2+4)}$$

$$= \frac{2}{(s-2)(s^2-4s+8)}$$

2. $L\{e^{t}(\sin t+\cos t)\}$

Sol :- $L\{\sin t+\cos t\} = \frac{1}{s^2+1} + \frac{s}{s^2+1} = \frac{1+s}{s^2+1} = f(s)$

$$\Rightarrow \left\{ \frac{e^{-s\pi}}{s^2+1} \left[0 - (-1) - (0-1) \right] \right.$$

$$\Rightarrow \frac{e^{-s\pi}}{s^2+1} \left[1+1 \right] \qquad \because \int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2} \left[a \sin bx - b \cos bx \right]$$

$$\Rightarrow \frac{2 \cdot e^{-s\pi}}{s^2+1} \Big\} \times$$

$$\Rightarrow \left[\frac{e^{-st}}{s^2+1} \left(-\sin t - \cos t \right) \right]_0^{\pi}$$

$$\Rightarrow \frac{e^{-s\pi}}{s^2+1} \left(-0 - (-1) \right) - \left[\frac{1}{s^2+1} \left(-0 - 1 \right) \right]$$

$$\Rightarrow \frac{e^{-s\pi}}{s^2+1} + \frac{1}{s^2+1}$$

(3)

$$\int_0^5 e^{-st} \cdot e^{t} \cdot dt + \int_5^{\infty} e^{-st} \cdot e^{2t} dt$$

$$\int_0^5 e^{-t(s-1)} dt + \int_5^{\infty} e^{-t(s-2)} dt$$

$$\Rightarrow \left[\frac{e^{-t(s-1)}}{-(s-1)} \right]_0^5 + \left[\frac{e^{-t(s-2)}}{-(s-2)} \right]_5^{\infty}$$

$$\Rightarrow \frac{e^{-5(s-1)}}{-(s-1)} + \frac{1}{+(s-1)} + 0 + \frac{e^{-5(s-2)}}{+(s-2)}$$

$$\Rightarrow \frac{e^{-5(s-1)}}{-(s-1)} + \frac{1}{(s-1)} + \frac{e^{-5(s-2)}}{(s-2)}$$

12. $L\{f(t)\}$ where (1) $f(t) = \begin{cases} 4, & 0<t<1 \\ 5, & t>1 \end{cases}$

(2) $f(t) = \begin{cases} \sin t, & 0<t<\pi \\ 0, & t \geq \pi \end{cases}$

(3) $f(t) = \begin{cases} e^{t}, & 0<t<5 \\ e^{2t}, & t \geq 5 \end{cases}$

Sol:- We Know that $L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$

$\Rightarrow$ (1) $4\int_0^1 e^{-st} \cdot dt + 5\int_1^\infty e^{-st} \cdot dt$

$\Rightarrow 4 \cdot \left[\frac{e^{-st}}{-s}\right]_0^1 + 5 \cdot \left[\frac{e^{-st}}{-s}\right]_1^\infty$

$\Rightarrow \frac{4}{-s}\left[e^{-s} - 1\right] + \frac{5}{-s}\left[0 - e^{-s}\right]$

$\Rightarrow \frac{-4e^{-s}}{s} + \frac{4}{s} + \frac{5e^{-s}}{s}$

$\Rightarrow \frac{e^{-s}}{s} + \frac{4}{s}$

(2) $\cancel{\sin t \cdot \int_0^\pi e^{-st}\, dt + 0\int_\pi^\infty e^{-st} \cdot dt}$

$\Rightarrow \sin t \cdot \left[\frac{e^{-st}}{-s}\right]_0^\pi + 0 \left[\frac{e^{-st}}{-s\pi}\right]_0^\infty$

$\Rightarrow \frac{\sin t}{-s}\left[e^{-s\pi} - 1\right] + 0$

$\Rightarrow \frac{-1}{s}\sin t \cdot e^{-s\pi} + \frac{1}{s}\}$ X

$\int_0^\pi e^{-st} \cdot \sin t\, dt + \int_0^\infty e^{-st} \cdot 0\, dt$

$\Rightarrow \frac{e^{-st}}{\ldots}\left[\ldots \cos t\right]^\pi$

$\frac{1}{2}\left[L\{\sin 5t\} - L\{\sin t\}\right]$

$\Rightarrow \frac{1}{2}\left[\frac{5}{s^2+25} - \frac{1}{s^2+1}\right]$

$= \frac{1}{2}\left[\frac{5s^2+5-s^2-25}{(s^2+25)(s^2+1)}\right]$

$= \frac{1}{2}\left[\frac{-20+4s^2}{(s^2+1)(s^2+25)}\right]$

10. $L\{(t^2+1)^2\}$

Sol:- $L\{t^4+2t^2+1\}$

$\Rightarrow L\{t^4\} + 2L\{t^2\} + 1L\{1\}$.

$\Rightarrow \frac{4!}{s^{4+1}} + \frac{2\cdot 2!}{s^{2+1}} + \frac{1}{s}$

$\Rightarrow \frac{24}{s^5} + \frac{4}{s^3} + \frac{1}{s}$

$\Rightarrow \frac{24+4s^2+s^4}{s^5}$

11. $L\{t^4+t^{3/2}+4\}$

Sol:- $L\{t^4\} + L\{t^{3/2}\} + L\{4\}$

$\Rightarrow \frac{4!}{s^{4+1}} + \frac{4}{s} + \frac{\Gamma(3/2+1)}{s^{3/2+1}}$

$\Rightarrow \frac{24}{s^5} + \frac{4}{s} + \frac{\Gamma(5/2)}{s^{5/2}}$

$\Rightarrow \frac{24}{s^5} + \frac{4}{s} + \frac{\frac{3}{2}\cdot\frac{1}{2}\Gamma(1/2)}{s^{5/2}}$

$\Rightarrow \frac{24}{s^5} + \frac{4}{s} + \frac{3\sqrt{\pi}}{4s^{5/2}}$

6. $L\{\sinh^2 at\}$

Sol: $L\left\{\frac{1-\cosh 2at}{2}\right\}$

$\Rightarrow \frac{1}{2}\left[L\{1\} - L\{\cosh 2at\}\right]$

$\Rightarrow \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2-4a^2}\right]$

$\Rightarrow \frac{1}{\cancel{2}}\left[\frac{\cancel{s^2}-\cancel{4}a^2-\cancel{s^2}}{s(s^2-4a^2)}\right]$

$\Rightarrow \frac{-2a^2}{s(s^2-4a^2)}$.

7. $L\{\sin^3 t\}$

$\sin 3A = 3\sin A - 4\sin^3 A$

Sol: $L\left\{\frac{3\sin t - \sin 3t}{4}\right\}$

$\therefore \sin^3 A = \frac{3\sin A - \sin 3A}{4}$

$\Rightarrow \frac{1}{4}\left[L\{3\sin t\} - L\{\sin 3t\}\right]$

$\Rightarrow \frac{1}{4}\left[\frac{3.(1)}{s^2+1} - \frac{3}{s^2+9}\right]$

$\Rightarrow \frac{1}{4}\left[\frac{3\cancel{s^2}+27-\cancel{3s^2}+3}{(s^2+1)(s^2+9)}\right]$

$\Rightarrow \frac{1}{2}\frac{(15)}{(s^2+1)(s^2+9)}$

SA

8. $L\{\cosh^2 at\}$

$\cosh^2 A$

Sol: Formula.

$\cosh 2A = 1-$

9. $L\{\sin 2t \cos 3t\}$

Sol: $\frac{1}{2}\frac{L}{\cancel{2}}\{2\sin 2t . \cos 3t\}$

$\Rightarrow \frac{1}{2}\ \{\sin(5t) + \sin(-t)\}$

$$\Rightarrow \frac{\omega \cdot \cos\alpha}{s^2+\omega^2} \pm \frac{s \sin\alpha}{s^2+\omega^2}$$

$$\Rightarrow \frac{\omega \cdot \cos\alpha \pm s \cdot \sin\alpha}{s^2+\omega^2}$$

4. $L\{\sin^2 at\}$

Sol:- $L\left\{\frac{1-\cos 2at}{2}\right\}$

$$\Rightarrow \frac{1}{2}\left[L\{1\} - L\{\cos 2at\}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4a^2}\right]$$

$$\Rightarrow \frac{1}{\cancel{2}}\left[\frac{\cancel{s^2}+\cancel{4}^{2}a^2-\cancel{s^2}}{s(s^2+4a^2)}\right]$$

$$\Rightarrow \frac{1}{\cancel{2}}\left[\frac{2a^2}{s(s^2+4a^2)}\right]$$

5. $L\{\cos^3 t\}$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\Rightarrow \cos^3 A = \frac{\cos 3A + 3\cos A}{4}$

Sol:- $L\left\{\frac{\cos 3t + 3\cos t}{4}\right\}$

$$\Rightarrow \frac{1}{4}\left[L\{\cos 3t\} + 3L\{\cos t\}\right]$$

$$\Rightarrow \frac{1}{4}\left[\frac{s}{s^2+9} + \frac{3 \cdot s}{s^2+1}\right]$$

$$\Rightarrow \frac{1}{4}\left[\frac{s^3+s+3s^3+27s}{(s^2+9)(s^2+1)}\right]$$

$$\Rightarrow \frac{1}{4}\left[\frac{4s^3+28s}{(s^2+9)(s^2+1)}\right]$$

$$\Rightarrow \frac{s^3+7s}{(s^2+9)(s^2+1)}$$

4/January/2017

1. $L\left\{\dfrac{e^{at}-1}{a}\right\}$

sol:- $\dfrac{1}{a}\left[L\{e^{at}\}-L\{1\}\right]$

$= \dfrac{1}{a}\left[\dfrac{1}{s-a}-\dfrac{1}{s}\right]$

$= \dfrac{1}{a}\left[\dfrac{s-(s-a)}{(s-a)(s)}\right]$

$= \dfrac{1}{\cancel{a}}\left[\dfrac{\cancel{s}-\cancel{s}+\cancel{a}}{s(s-a)}\right]$

$= \dfrac{1}{s(s-a)}$

2. $L\{(\sin t-\cos t)^2\}$

sol:- $L\{1-\sin 2t\}$

$\Rightarrow L\{1\} - L\{\sin 2t\}$

$\Rightarrow \dfrac{1}{s} - \dfrac{2}{(s^2+4)}$

$\Rightarrow \dfrac{s^2+4-2s}{s(s^2+4)}$

$\Rightarrow \dfrac{s^2-2s+4}{s(s^2+4)}$

3. $L\{\sin(\omega t \pm \alpha)\}$

sol:- $L\{\sin \omega t \cos\alpha \pm \cos\omega t \sin\alpha\}$

$\Rightarrow \cos\alpha \, L\{\sin \omega t\} \pm \sin\alpha \, L\{\cos\omega t\}$

$\Rightarrow \cos\alpha \left(\dfrac{\omega}{s^2+\omega^2}\right) \pm \sin\alpha\left(\dfrac{s}{s^2+\omega^2}\right)$

(n-2)
x)(n-x
so
ration

Find $L\{te^{t}\sin t\}$.

$L\{\sin 1t\} = \frac{1}{s^2+1} = f(s)$

M by t

$\{t\sin t\} = (-1)^1 \frac{d}{ds}\left[\frac{1}{s^2+1}\right]$

$= (-1)\,\frac{(-1)}{(s^2+1)^2} \times 2s$

$\{t\sin t\} = \frac{2s}{(s^2+1)^2}$

By F.S. Theorem.

$L\{e^{1t}(-\sin t)\} = \frac{2(s-1)}{((s-1)^2+1)^2}$

$= \frac{2(s-1)}{s^2+2s+2}$

$\int_0^\infty e^{t}\cdot t\cdot \sin t\, dt$

W.K.T

$L\{t\sin t\} = \frac{2s}{(s^2+1)^2}$

$\int_0^\infty e^{-st}\cdot t\cdot \sin t\, dt = \frac{2s}{(s^2+1)^2}$

Put $s = -1$

$\int_0^\infty e^{1t}\cdot t\cdot \sin t\cdot dt = \frac{-2}{(1+1)^2}$

$= -2/4 = -1/2$

Find $L\{te^{-3t}\cos t\}$

$\{\cos t\} = \frac{s}{s^2+1} = f(s)$

M by t

$\{t\cos t\} = (-1)^1 \frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$= (-1)\cdot\left[\frac{(s^2+1)(1) - s\cdot 2s}{(s^2+1)^2}\right]$

$L\{t\cos t\} = \frac{(-1)(1-s^2)}{(s^2+1)^2} = \frac{s^2-1}{(s^2+1)^2}$

By F.S. Theorem

$L\{e^{-3t}\,t\cdot\cos t\} = \frac{(s+3)^2-1}{[(s+3)^2+1]^2}$

$\int_0^\infty e^{-3t}\cdot t\cdot \cos t\cdot dt$

W.K.T

$L\{t\cos t\} = \frac{s^2-1}{(s^2+1)^2}$

$\int_0^\infty e^{-st}\cdot t\cdot \cos t\cdot dt = \frac{s^2-1}{(s^2+1)^2}$

Put $s = 3$

$\int_0^\infty e^{-3t}\cdot t\cos t\cdot dt = \frac{8}{100}$

$= \frac{4}{50}$

$$\frac{e^{-at}\sin^2 t}{t}\,dt$$

$$L\{\sin^2 t\} = L\left\{\frac{1-\cos 2t}{2}\right\}$$

$$\frac{1}{2}\left[L\{1\} - L\{\cos 2t\}\right]$$

$$\frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right] = f(s)$$

D by t

$$L\left\{\frac{\sin^2 t}{t}\right\} = \frac{1}{2}\int_s^\infty \left[\frac{1}{s} - \frac{s}{s^2+4}\right] ds.$$

$$= \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2+4)\right]_s^\infty$$

$$L\left\{\frac{\sin^2 t}{t}\right\} = \frac{1}{2}\left[\log s - \log\sqrt{s^2+4}\right]_s^\infty$$

$$= \frac{1}{2}\log\left[\frac{s}{\sqrt{s^2+4}}\right]_s^\infty = \frac{1}{2}\left[\lim_{s\to\infty}\log\left(\frac{s(1)}{s\sqrt{1+\frac{4}{s^2}}}\right) - \log\left(\frac{s}{\sqrt{s^2+4}}\right)\right]$$

$$= \frac{1}{2}\left[0 - \log\left(\frac{s}{\sqrt{s^2+4}}\right)\right] = -\frac{1}{2}\log\left(\frac{s}{\sqrt{s^2+4}}\right)$$

$$\int e^{-st}\left(\frac{\sin^2 t}{t}\right)dt = -\frac{1}{2}\log\left(\frac{s}{\sqrt{s^2+4}}\right)$$

Put $s = a$.

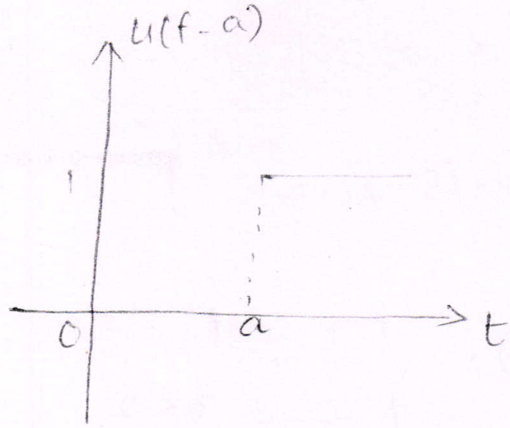
$$\int e^{-at}\left(\frac{\sin^2 t}{t}\right)dt = -\frac{1}{2}\log\left(\frac{a}{\sqrt{a^2+4}}\right)$$

UNIT step fn / Heaviside's Unit function:

$u(t-a) = u_a(t)$

$$H(t-a) = \begin{cases} 1, & t>a \\ 0, & t<a \end{cases}$$

where $a>0$



Ex: $$L\{u(t-a)\} = \int_0^\infty e^{-st}\, u(t-a)\, dt$$

$$= \int_0^a e^{-st}\cdot 0 \cdot dt + \int_a^\infty e^{-st}\cdot 1 \cdot dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$= \frac{0 - e^{-as}}{-s}$$

$$L\{u(t-a)\} = \frac{e^{-as}}{s}$$

Ily $$L\{u(t)\} = \frac{1}{s}$$

$$L\{u(t-a)\cdot f(t-a)\} = e^{-as}\cdot f(s) = e^{-as}\cdot L\{f(t)\}$$

Find $$L\{5u(t-4)\cdot \sin(t-4)\} = 5e^{-4s}\cdot L\{\sin 1t\}$$

$$= \frac{5e^{-4s}}{s^2+1}$$

$$L\{u(t-2)e^{(t-2)}\}$$

W.K.T $\quad L\{u(t-2)\cdot e^{t-2}\}$

W.K.T

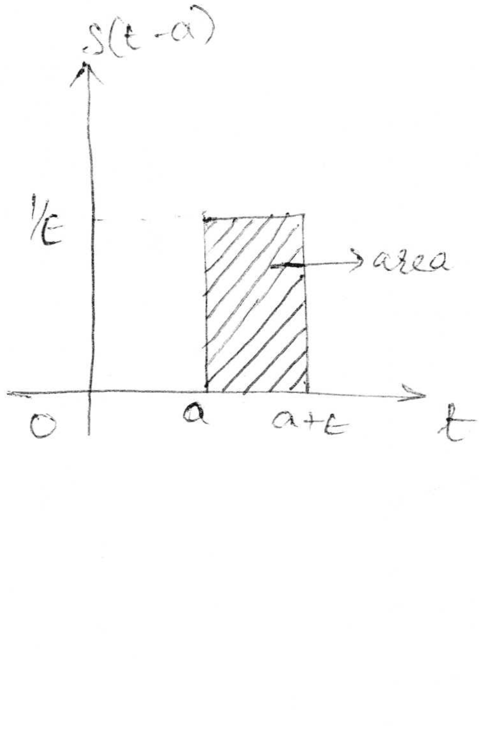
$$u(t-2) = \begin{cases} 1, & t > 2 \\ 0, & t < 2 \end{cases}$$

$$L\{u(t-2)\} = \frac{e^{-2s}}{s}$$

~~$L\{t\,e$~~

Unit Impulse Function :

$$\delta(t-a) = \begin{cases} \lim_{\epsilon \to 0} \frac{1}{\epsilon}, & a \le t \le a+\epsilon \\ 0, & \text{otherwise} \end{cases}$$



$$\delta(t-a) = \begin{cases} 1/0, & a \le t \le a \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(t-a) = \begin{cases} \infty, & t = a \\ 0, & \text{otherwise} \end{cases}$$

$$\int \delta(t-a)\,dt = 1$$

Periodic function :

A function which Repeats itself with a Period T>0 is Known as Periodic function i.e.

$f(x) = f(x) + T = f(x + 2t)$.

T → Period.

Period of $\sin x = 2\pi$

" " $\cos x = 2\pi$

" " $\tan x = \pi$

" " $\cot x = \pi$

→ If f(t) is a periodic function with a period T then

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\,dt}{1 - e^{-sT}}$$

Find $L\{f(t)\}$ where f(t) is a periodic function of period 2π and is given by $f(t) = \begin{cases} \sin t, & 0<t<\pi \\ 0, & 0<t<2\pi \end{cases}$

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\,dt}{1 - e^{-sT}}$$

$$= \frac{1}{1 - e^{-2s\pi}} \int_0^{2\pi} e^{-st} f(t)\,dt$$

$$L\{f(t)\} = \frac{1}{1 - e^{-2s\pi}} \left[\int_0^{\pi} e^{-st} \sin 1t\,dt + 0 \right]$$

$$\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b\cos bx)$$

$$= \frac{1}{1 - e^{-2s\pi}} \left[\frac{e^{-st}}{s^2+1} (-s\sin t - \cos t) \right]_0^{\pi}$$

$$\frac{1}{1-e^{-2\pi s}}\left[\frac{e^{-s\pi}}{s^2+1}(0-(-1))-\frac{1}{s^2+1}(0-1)\right]$$

$$\frac{(e^{-s\pi}+1)}{(1-e^{-2\pi s})(s^2+1)}$$

$$=\frac{(e^{-s\pi}+1)}{(1+e^{-s\pi})(1-e^{-s\pi})(s^2+1)}$$

$$=\frac{1}{(s^2+1)(1-e^{-s\pi})}$$

Find the laplace transform of the square wave for period $2a$ defined as $f(t)=\begin{cases}K, & 0<t<a\\ -K, & a<t<2a\end{cases}$

$$L\{f(t)\}=\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$$

$$=\frac{1}{1-e^{-2as}}\left[\int_0^{a} e^{-st}K\,dt+\int_a^{2a}-e^{-st}K\,dt\right]$$

$$=\frac{1}{1-e^{-2as}}\left[K\left[\left(\frac{-e^{-st}}{s}\right)_0^a-\left(\frac{e^{-st}}{-s}\right)_a^{2a}\right]\right]$$

$$=\frac{K}{1-e^{-2as}}\left[\left[\frac{-e^{-sa}}{s}+\left(\frac{1}{s}\right)\right]-\left[\frac{-e^{-2sa}}{s}+\frac{e^{-sa}}{s}\right]\right]$$

$$=\frac{K}{1-e^{-2as}}\left[\frac{-2e^{-sa}}{s}+\frac{1}{s}+\frac{e^{-2sa}}{s}\right]$$

$$= \frac{K}{1-e^{-2as}}\left[\frac{-e^{-2sa}+1-2e^{-sa}}{s}\right]$$

$$= \frac{K}{1-e^{-2as}}\left[\frac{(e^{-sa}-1)^2}{s}\right]$$

$$= \frac{K}{(1+e^{-as})\cancel{(1-e^{-as})}}\left[\frac{(e^{-sa}-1)^{\cancel{2}}}{s}\right]$$

$$= \frac{-K(e^{-sa}-1)}{s(1+e^{-as})}.$$

$$= \frac{K(1-e^{-sa})}{s(1+e^{-as})}$$

(x)

→ Find $L^{-1}\left\{\frac{4}{(s+1)(s+2)}\right\}$

$$\frac{4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}.$$

$$\frac{4}{(s+1)(s+2)} = \frac{A(s+2) + B(s+1)}{(s+1)(s+2)}$$

$4 = A(s+2) + B(s+1).$

Comparing coeff's of s

$A + B = 0$

$A = -B$

Comparing coeff's of Const

$4 = 2A + B.$

$4 = 2A - A$

$$\boxed{\begin{array}{l} A = 4 \\ B = -4 \end{array}}$$

$$= L^{-1}\left\{\frac{4}{s+1}\right\} - 4L^{-1}\left\{\frac{1}{s+2}\right\}.$$

$$= 4e^{-t} - 4e^{-2t}$$

→ Find $L^{-1}\left\{\frac{1}{s(s+1)(s+2)}\right\}.$

$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}.$$

$1 = A(s+1)(s+2) + B(s)(s+2) + C(s)(s+1).$

1) $1 = B(-1)(1) \Rightarrow B = -1.$ $s=0$ $1 = A(1)(2)$

$A = 1/2$

2)

$$\left\{\frac{1}{2s}\right\} + L^{-1}\left\{\frac{-1}{s+1}\right\} + L^{-1}\left\{\frac{1}{2(s+2)}\right\}$$

$$\left\{\frac{1}{s}\right\} + L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s+2}\right\}$$

$$(1) \quad - e^{-t} + \frac{1}{2} e^{-2t}$$

$$L^{-1}\left\{\frac{s^2+s-2}{s(s+3)(s-2)}\right\}$$

$$\frac{s^2+s-2}{s(s+3)(s-2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-2}$$

$$s^2+s-2 = A(s+3)(s-2) + B(s)(s-2) + C(s)(s+3)$$

$$-\cancel{2} = A(3)(-\cancel{2})$$

$$\boxed{A = 1/3}$$

$$\cancel{s}^2 + \cancel{2} - \cancel{2} = C(\cancel{2})(5)$$

$$\boxed{C = 2/5}$$

$$9-3-2 = B(-3)(-5)$$

$$4 = B(15)$$

$$\boxed{B = 4/15}$$

$$L^{-1}\left\{\frac{1}{s}\right\} + \frac{4}{15} L^{-1}\left\{\frac{1}{s+3}\right\} + \frac{2}{5} L^{-1}\left\{\frac{1}{s-2}\right\}$$

$= \frac{1}{3}(1) + \frac{4}{15} e^{3t} + \frac{2}{5} e^{-2t}$

$= \frac{1}{3} + \frac{4}{15} e^{-3t} + \frac{2}{5} e^{+2t}$

Find $L^{-1}\left\{\frac{s^2+2s-4}{(s^2+9)(s-5)}\right\}$

$$\frac{s^2+2s-4}{(s^2+9)(s-5)} = \frac{As+B}{s^2+9} + \frac{C}{s-5}$$

$$s^2+2s-4 = As^2 + sB - 5As - 5B + Cs^2 + 9C$$

Comp s^2

$1 = A + C$

$C = 1 - \frac{3}{34}$

$\boxed{C = \frac{31}{34}}$

s

$2 = B - 5A$

$5\times$ $\cancel{10 = 5B - 25A}$

$\cancel{-4 = -5B + 9C}$

$\cancel{6 = -25A}$

$B = 2 + 5A$

$B = 2 + 5\left(\frac{3}{34}\right)$

$\boxed{B = \frac{83}{34}}$

Const

$-4 = -5B + 9C$

$-4 = -5(2+5A) + 9C$

$-4 = -10 - 25A + 9C$

$6 = -25A + 9C$

$9 = 9A \pm 9C$

$-3 = 34A$

$\boxed{A = 3/34}$

$\frac{31}{34} L^{-1}\left\{\frac{3s+83}{s^2+9}\right\}$

$$\frac{3}{34} L^{-1}\left\{\frac{s}{s^2+3^2}\right\} + \frac{83}{34} L^{-1}\left\{\frac{1}{s^2+3^2}\right\} + \frac{31}{34} L^{-1}\left\{\frac{1}{s-5}\right\}.$$

$$= \frac{3}{34}\cos 3t + \frac{83}{34}\cdot\frac{1}{3}\sin 3t + \frac{31}{34} e^{5t}$$

Find $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+25)}\right\}$.

$$\frac{}{)(s^2+25)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+25}$$

$$= As^3+Bs^2+25As+25B+Cs^3+Ds^2+4Cs+4D$$

$-C=0$

$A=-C$

$C=0$

$1=B+D$

$B=1-D$

$B=1-\frac{25}{21}$

$B=-\frac{4}{21}$

$25A+4C=0$

$25A-4A=0$

$21A=0$

$A=0$

$25B+4D=0$

$25(1-D)+4D=0$

$-21D+25=0$

$D=\frac{+25}{21}$

$$-\frac{4}{21}L^{-1}\left\{\frac{1}{s^2+4}\right\} + \frac{25}{21}L^{-1}\left\{\frac{1}{s^2+25}\right\}$$

$$-\frac{4}{21}\times\frac{1}{2}\sin 2t + \frac{25}{21}\times\frac{1}{5}\times\sin 5t$$

$$-\frac{2}{21}\sin 2t + \frac{5}{21}\sin 5t$$

Find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+9)(s^2+25)}\right\}$

$$\frac{}{{}^2+9)(s^2+25)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9} + \frac{Es+F}{s^2+25}$$

~~find L^{-1}~~

F.S.T for inverse laplace transformation:

If $L^{-1}\{f(s)\} = f(t)$

then

(a) $L^{-1}\{f(s-a)\} = e^{at}\cdot f(t)$

(b) $L^{-1}\{f(s+a)\} = e^{-at}\cdot f(t)$.

find $L^{-1}\left\{\frac{4}{(s+1)^2(s+3)}\right\}$.

$$\frac{4}{(s+1)^2(s+3)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+3}.$$

$$4 = A(s+1)(s+3) + B(s+3) + C(s+1)^2$$

Comp s^2 Coeff:	Comp s Coeff	Comp Const or Put $s=0$
$0 = A + C$	$0 = 4A + B + 2C$	$4 = 3A + 3B + C$

$$A = -1, \quad B = 2, \quad C = 1$$

$$L^{-1}\left\{\frac{4}{(s+1)^2(s+3)}\right\} = -L^{-1}\left\{\frac{1}{s+1}\right\} + 2L^{-1}\left\{\frac{1}{(s+1)^2}\right\} + L^{-1}\left\{\frac{1}{s+3}\right\}.$$

$$= -e^{-t} + e^{-3t} + 2e^{-t}\cdot L^{-1}\left\{\frac{1}{s^2}\right\}.$$

$$= -e^{-t} + e^{-3t} + 2e^{-t}\cdot t.$$

→ find $L^{-1}\left\{\frac{s+3}{(s+1)^2(s+3)}\right\}$

$$\frac{3}{)^2(s+3)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+3}$$

$$s+3 = A(s+1)(s+3) + B(s+3) + C(s+1)^2$$

$$3 = A(s^2+4s+3) + B(s+3) + C(s^2+2s+1)$$

$+C = 0$ | $4A+B+2C = 1$ | $3A+3B+C = 3$

$A = -C$ | $2A+B = 1$ | $2A+3B = 3$

$2A+B = 1$

$2B = 2$

$B = 1$

$2A = 1-1$

$A = 0$

$C = 0$

$$L^{-1}\left\{\frac{1}{(s+1)^2}\right\} = e^{-t} \cdot L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$= e^{-t} \cdot t$$

$$L^{-1}\left\{\frac{3}{s^2+2s+3}\right\}$$

$$= L^{-1}\left\{\frac{3}{(s+1)^2+2}\right\}$$

$$= 3L^{-1}\left\{\frac{1}{(s+1)^2+(\sqrt{2})^2}\right\}$$

s^2+2s+1 $$= 3e^{-t}L^{-1}\left\{\frac{1}{s^2+(\sqrt{2})^2}\right\}$$

$$= \frac{3e^{-t}}{\sqrt{2}} \sin\sqrt{2}t$$

$\rightarrow L^{-1}\left\{\dfrac{s}{s^2+2s+3}\right\}$

$$= L^{-1}\left\{\frac{(s+1)-1}{(s+1)^2+(\sqrt{2})^2}\right\}$$

$$= e^{-t}\cdot L^{-1}\left\{\frac{s-1}{s^2+(\sqrt{2})^2}\right\}$$

$$= e^{-t}\left[\cos\sqrt{2}t - 1/\sqrt{2}\,\sin\sqrt{2}t\right].$$

$\Rightarrow$ e^{-1} G

Second shifting theorem:

If $L^{-1}\{f(s)\} = f(t)$ then $L^{-1}\{e^{-as} f(s)\} = G(t)$

where $G(t) = \begin{cases} f(t-a), & t>a \\ 0, & t<a \end{cases}$

$$L^{-1}\left\{\frac{e^{-4s}}{s^2+4}\right\}.$$

W.K.T

$$L^{-1}\left\{\frac{1}{s^2+2^2}\right\} = \frac{1}{2}\sin 2t$$

By S.S. theorem

$$L^{-1}\left\{\frac{e^{-4s}\times 1}{s^2+4}\right\} = \begin{cases} \dfrac{\sin 2(t-4)}{2}, & t>4 \\ 0, & t<4 \end{cases}$$

$$= \frac{1}{2}\sin(2t-8)\cdot H(t-4).$$

$$L^{-1}\left\{\frac{1+e^{-\pi s}}{s^2+1}\right\}$$

$$L^{-1}\left\{\frac{1}{s^2+1}\right\} + L^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}.$$

$$\sin t + \sin(t-\pi)\cdot H(t-\pi)$$

$$L^{-1}\left\{\frac{e^{-3s}}{(s-4)^2}\right\}$$

$$e^{4t}\cdot L^{-1}\{e^{-3}$$

$$L^{-1}\left\{\frac{1}{(s-4)^2}\right\} = e^{4t}\cdot L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$= t\cdot e^{4t}$$

By s s theorem.

$$L^{-1}\left\{\frac{e^{-3s}}{(s-4)^2}\right\} = (t-3)\, e^{4(t-3)}\cdot H(t-3).$$

$$L^{-1}\left\{\frac{e^{-2s}}{s^2+4s+5}\right\}$$

$$\Rightarrow L^{-1}\left\{\frac{e^{-2s}}{(s+2)^2+1}\right\}$$

$$= e^{-2t}\cdot L^{-1}\left\{\frac{e^{-2s}}{s^2+1^2}\right\}$$

5 -

=

C

0

$= e^{-2t} \sin t$

By S.S Theorem

$$L^{-1}\left\{\frac{e^{-2s}}{s^2+4s+5}\right\} = e^{-2(t-2)} \cdot \sin(t-2) \times H(t-2).$$

$$\Rightarrow L^{-1}\left\{\frac{3+5s}{s^2 e^{2s}}\right\}.$$

$$= L^{-1}\left\{e^{-2s} \cdot \left(\frac{3+5s}{s^2}\right)\right\}$$

$$= L^{-1}\left\{\frac{3+5s}{s^2}\right\}.$$

$$= 3t+5$$

By S.S. Theorem.

$$\Rightarrow L^{-1}\left\{\frac{3+5s}{s^2 e^{2s}}\right\} = 3(t-2)+5 \times H(t-2).$$

<u>Change of Scale Properties:</u>

If $L^{-1}\{f(s)\} = f(t)$.

Then $L^{-1}\{f(as)\} = \frac{1}{a} f\left(\frac{t}{a}\right), a>0.$

NOTE:

$$L^{-1}\left\{\frac{as}{(a^2s^2+1)^2}\right\}.$$

$$= \frac{t}{2a^2} \sin\left(\frac{t}{a}\right).$$

$\left.\frac{s}{(s^2+1)^2}\right\} = \frac{t}{2}\sin t$

$$L^{-1}\left\{\frac{2s}{(4s^2+1)^2}\right\}.$$

$$= \frac{t}{8}\sin\left(\frac{t}{2}\right).$$

iplication by s:

If $L^{-1}\{f(s)\} = f(t)$ & $f(0)=0$ then

$\left. f(s)\right\} = f'(t)$ & $L^{-1}\{s^n f(s)\} = f^n(t)$ where

$=0,\ f'(0)=0, \dots f^{n-1}(0)=0$

$$L^{-1}\left\{\frac{s}{(s+2)^2}\right\}$$

$$= L^{-1}\left\{\frac{1}{(s+2)^2}\right\}.$$

$$= e^{-2t}\cdot L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$= e^{-2t}\cdot t.$$

s:

$$L^{-1}\left\{\frac{s}{(s+2)^2}\right\} = \frac{d}{dt}\left(e^{-2t}\cdot t\right) = \left[t\cdot(-2e^{-2t}) + e^{-2t}\right]$$

$$= -2te^{-2t} + e^{-2t}$$

$$= e^{-2t}[1-2t].$$

Inv

The

$\rightarrow$ f

i,

ii,

iii,

i,

L^{-1}

Inverse Laplace function of Derivatives :

If $L^{-1}\{f(s)\} = f(t)$

Then $L^{-1}\{f^{n}(s)\} = (-1)^n t^n f(t)$.

→ Find the Inverse transform of

i, $L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$

ii, $L^{-1}\left\{\log\left(\frac{s+3}{s+4}\right)\right\}$

iii, $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$.

i, $f(s) = \log\left(\frac{s^2+1}{s^2}\right)$.

$f(s) = \log(s^2+1) - \log(s^2)$

$f(s) = \log(s^2+1) - 2\log s$.

$f'(s) = \frac{1}{s^2+1}(2s) - \frac{2}{s}$.

$L^{-1}\{f'(s)\} = 2L^{-1}\left\{\frac{s}{s^2+1}\right\} - 2L^{-1}\left\{\frac{1}{s}\right\}$.

$(-1)^1 t^1 f(t) = 2\cos t - 2$

$f(t) = \frac{2(\cos t - 1)}{-t}$

$L^{-1} f(s) = \frac{2(1-\cos t)}{t}$

$\frac{s^2 - s^2}{s^2(s^2+a^2)}$

$\frac{1}{s^2}$ $\frac{1}{s^2+a^2}$

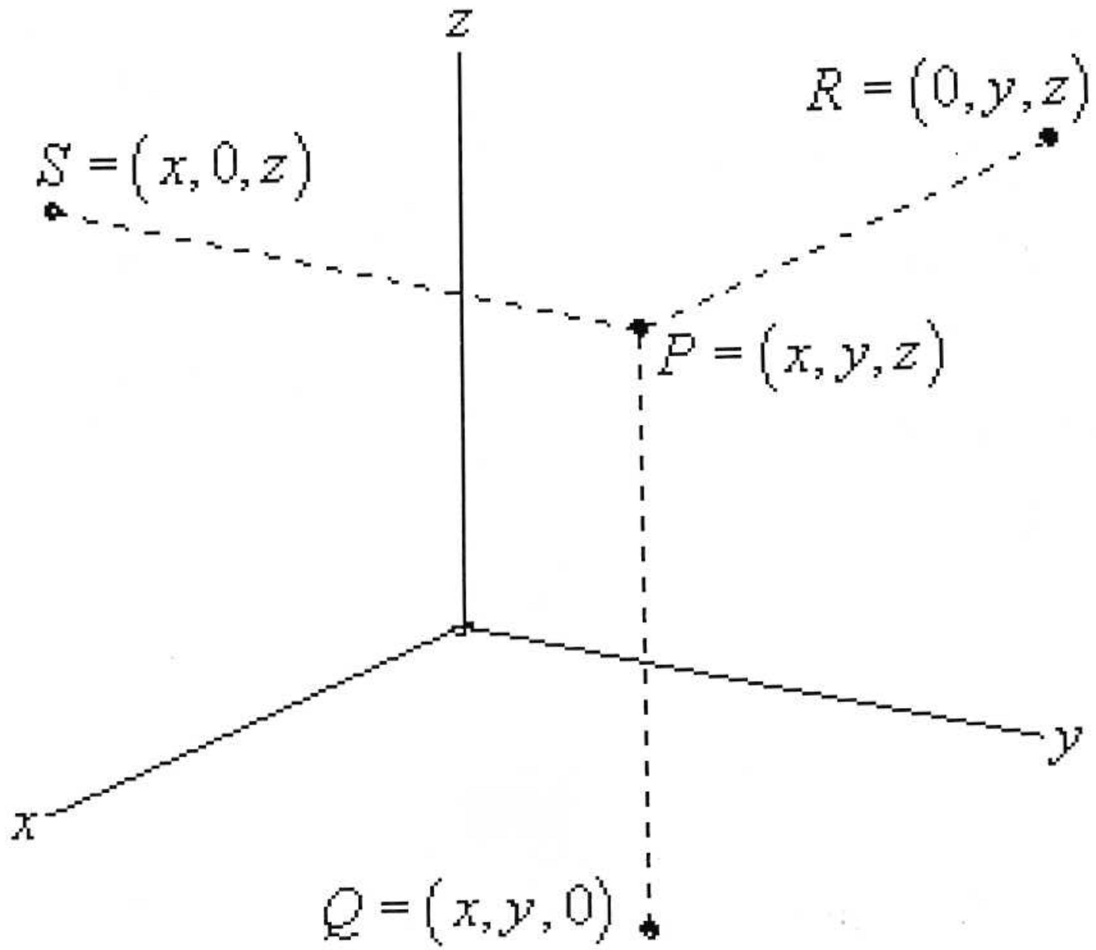
POWERPOINT PRESENTATION

Double and Triple Integrals

- Using Iterated Integrals to find area
- Using Double Integrals to find Volume
- Using Triple Integrals to find Volume

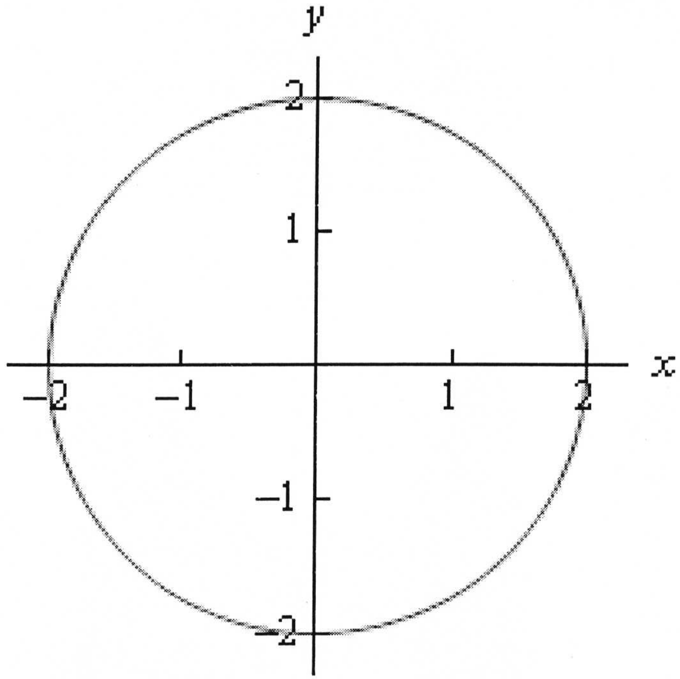
$$\iint_R f(x,y)\, dA$$

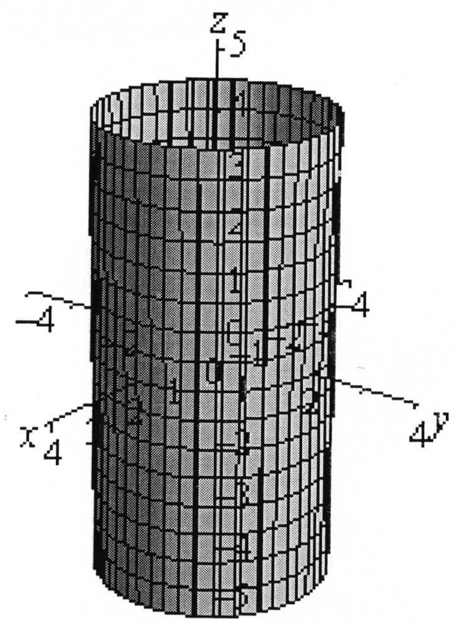
Three Dimensional Space



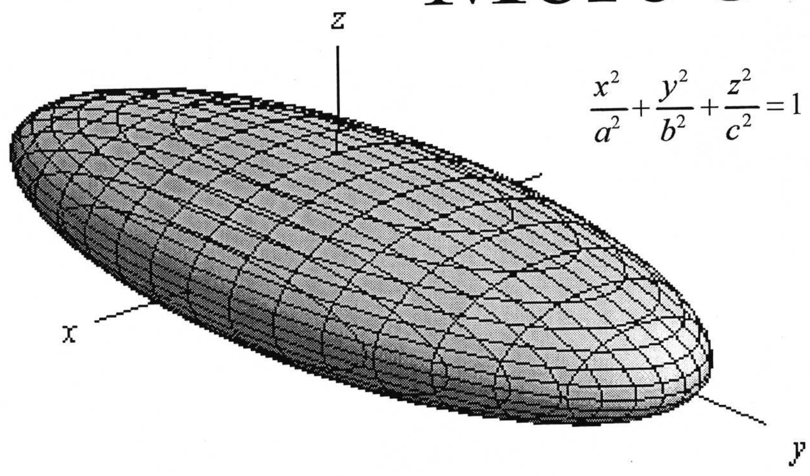
In Two-Dimensional Space, you have a circle

In Three-Dimensional space, you have a ______________!!!!!!!!!!!

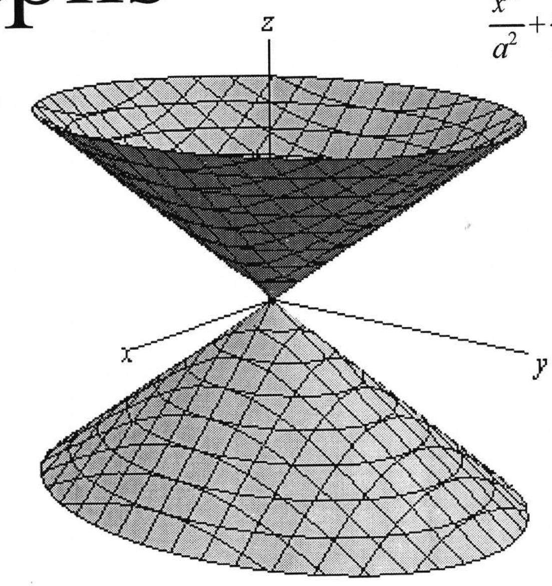




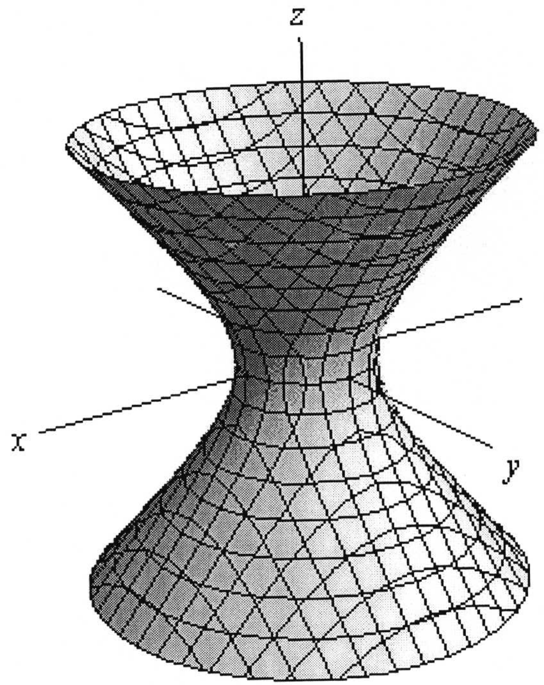
More 3-D graphs



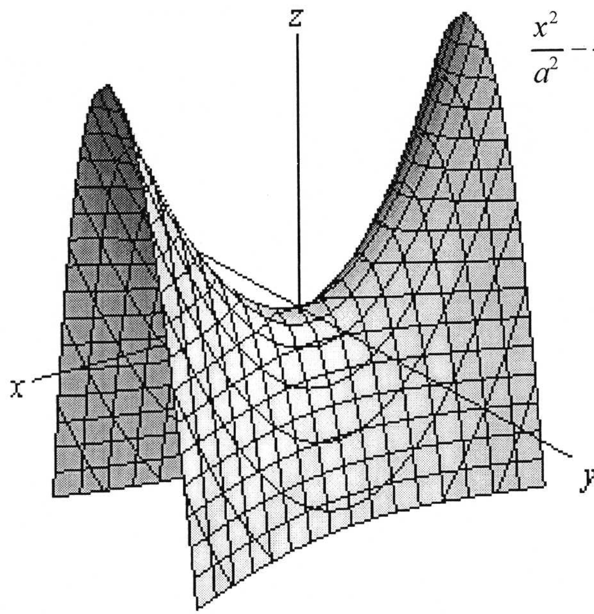
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

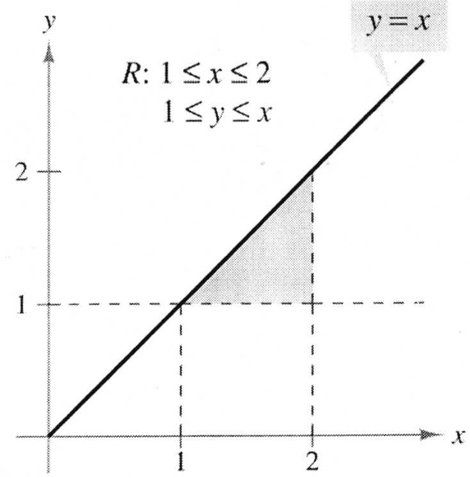


$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{z}{c}$$

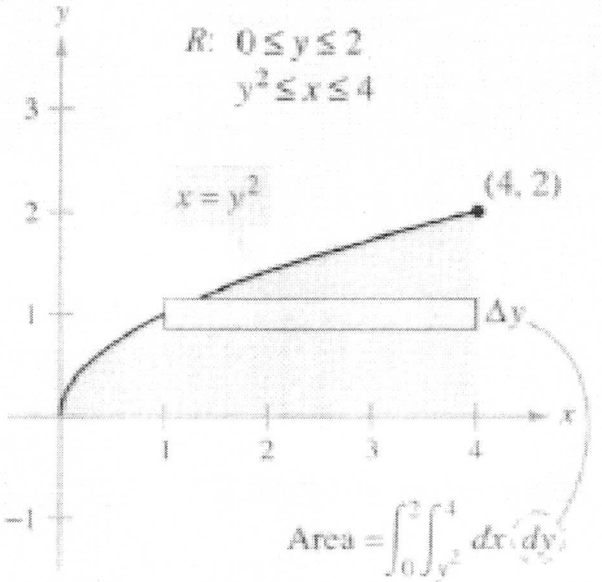
The Iterated Integral

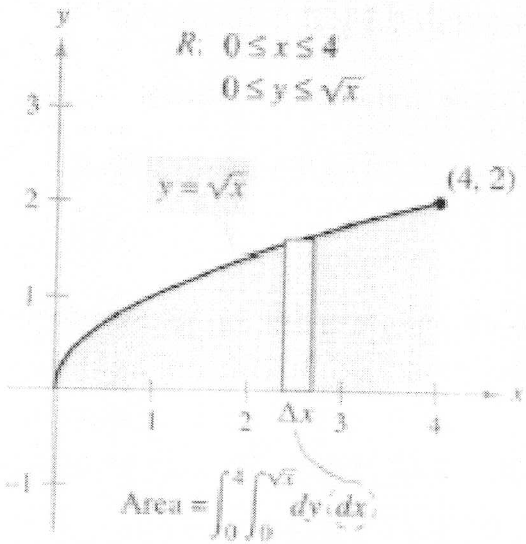
$$\int_{1}^{2y} 2xy\,dx$$

$$\int_{1}^{2}\int_{1}^{x} 2x^2y^{-2} + 2y\,dy\,dx$$

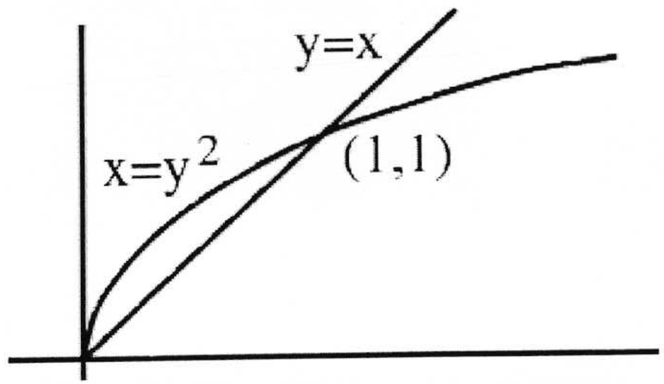


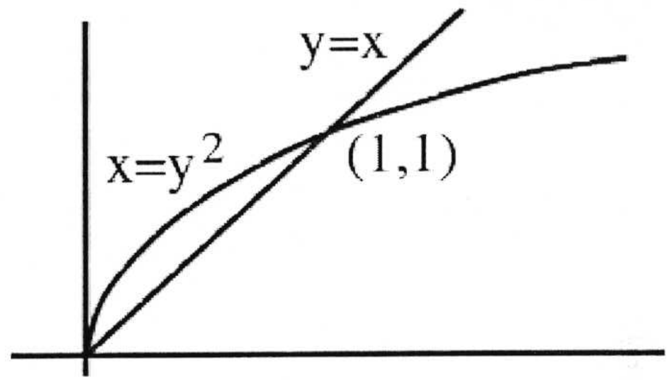
Setting up the Double Integral





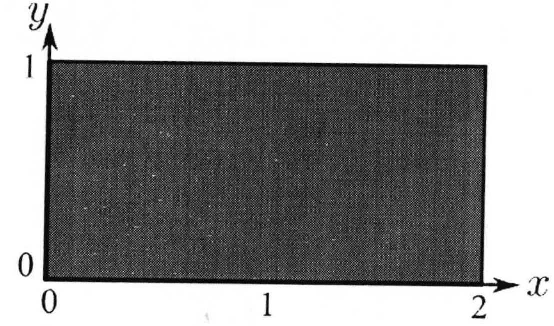
Finding Area using Double Integrals

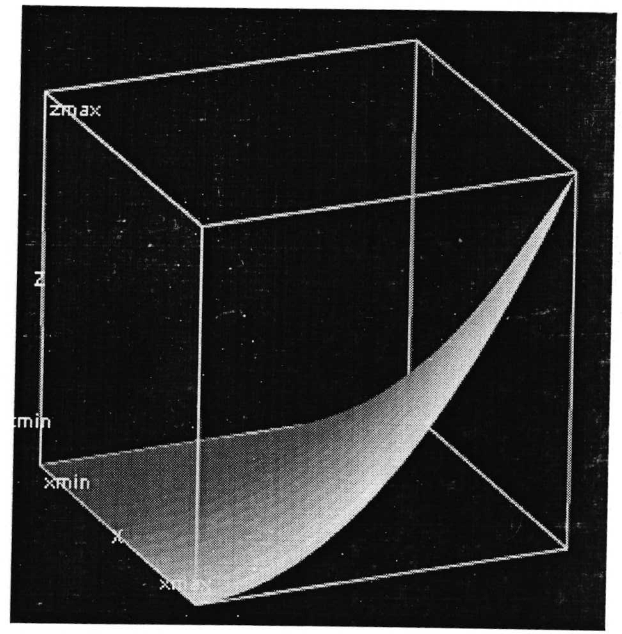




Compute the integral on the pictured region

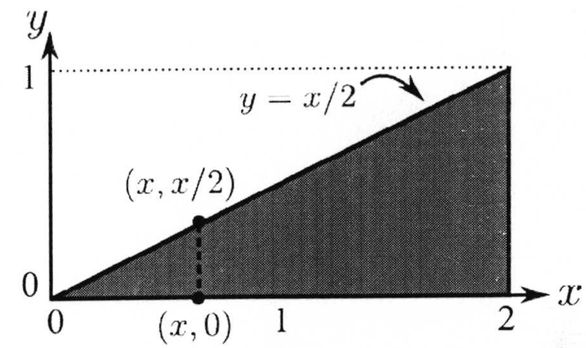
$$\iint_R xy^2\,dA$$



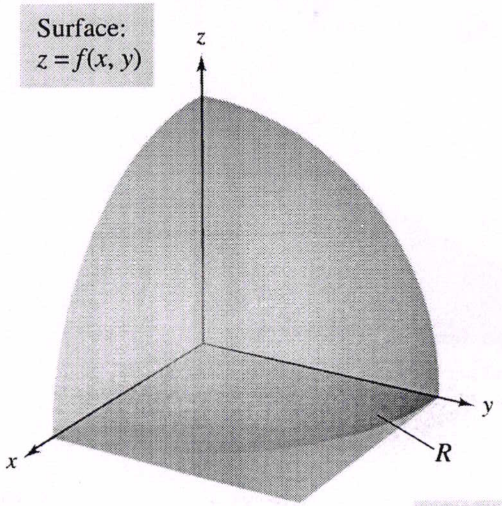


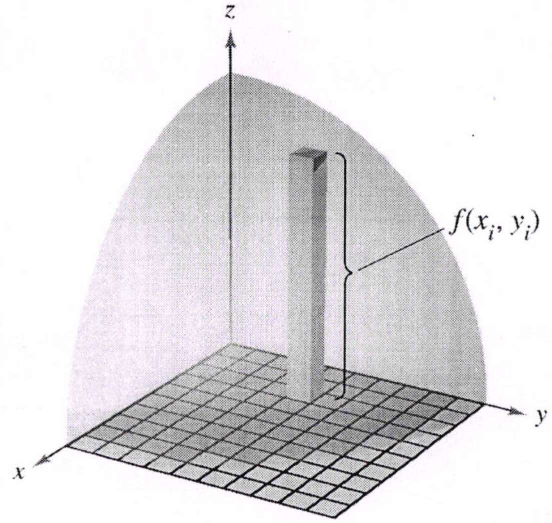
Compute the integral on the pictured region

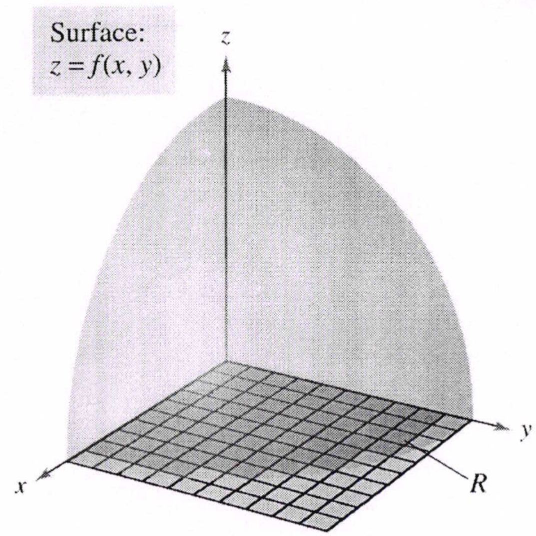
$$\iint_R xy^2\,dA$$

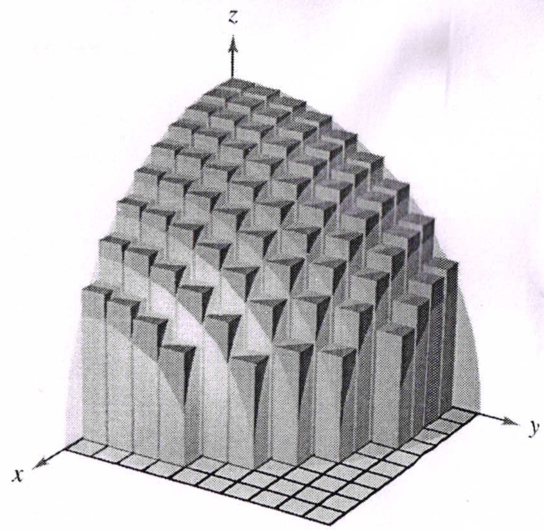


Finding Volume using the Double Integral







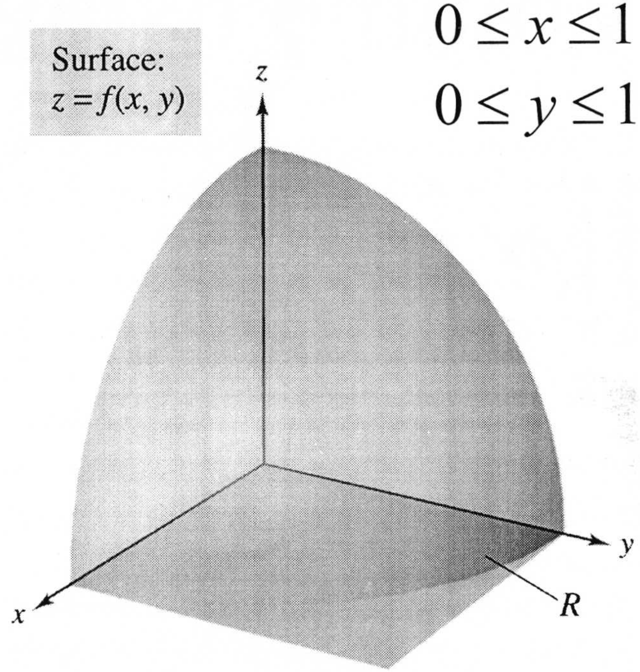


Evaluate the volume using the region

$$\iint_R 1-\frac{1}{2}x^2-\frac{1}{2}y^2\,dA$$

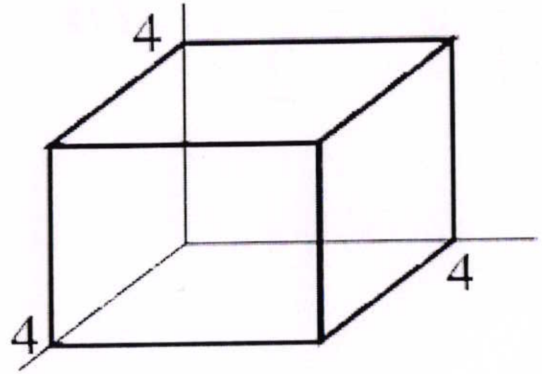
$0 \le x \le 1$

$0 \le y \le 1$



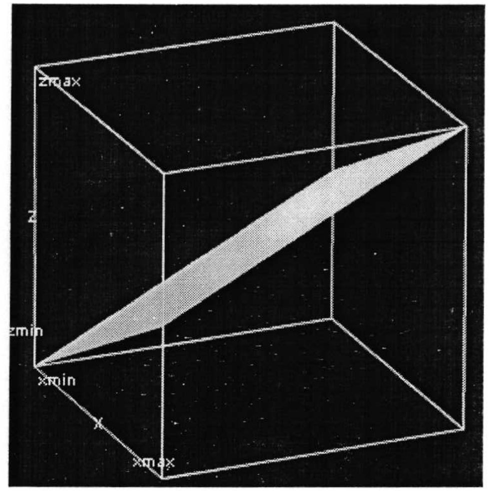
Volume using the Triple Integral

$$\int_0^4 \int_0^4 \int_0^4 dV$$



The cubes density is proportional to its distance away from the Xy-plane. Find its mass.

$$\int_0^2 \int_0^x \int_0^{1+x+y} dV$$



Solution of PDEs using the Laplace Transform*

- A powerful technique for solving ODEs is to apply the Laplace Transform
  - Converts ODE to algebraic equation that is often easy to solve
- Can we do the same for PDEs? Is it ever useful?
  - Yes to both questions – particularly useful for cases where periodicity ***cannot*** be assumed, thwarting use of Fourier series, hence separation of variables

*Kreysig, 8th Edn Sec 11.12

Laplace Transform

- The key point: we could handle functions that were discontinuous, in any event not periodic
- HLT has zillions of useful examples of LT:

$$v(t) \Rightarrow V(s)$$

$$W(s) \Rightarrow w(t)$$

We ask: can we apply the LT to solve ***Partial*** differential equations for cases where separation of variables is ineffective?

Example*: waggling a semi-infinite string

Find the displacement $w^{**}(x,t)$ of an elastic string subject to the following conditions:

1. The semi-infinite string is initially at rest, on the x-axis
2. For some time $t>0$ the left hand end of the string ($x=0$) is moved sinusoidally:

$$w(0,t) = \begin{cases} \sin t & \text{if } 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

3. Furthermore

$$\lim_{x \to \infty} w(x,t) = 0, \text{ for } t \ge 0$$

We will tackle this problem using the Laplace Transform; but first, we try a simpler example

*Kreysig, 8th Edn, page 644

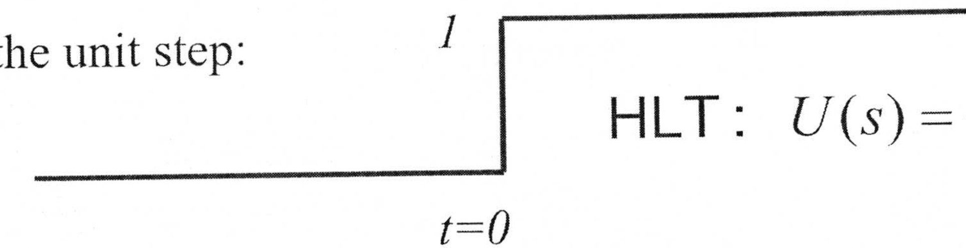
** just in this part of the notes, we use $w(x,t)$ for the PDE, rather than $u(x,t)$ because $u(t)$ is conventionally associated with the step function

A recap on the LT

We first solve the first order ODE

$$\dot{w}(t)+aw(t)=u(t) \qquad w(0)=1$$

where $u(t)$ is the unit step:



HLT : $U(s)=\frac{1}{s}$

Laplace Transform (reach for HLT): $(sW(s)-w(0))+aW(s)=\frac{1}{s}$

Rearranging, and partial fractions $W(s)=\frac{1}{as}+\frac{a-1}{a}\frac{1}{s+a}$

Once more unto HLT, inverse LT: $w(t)=\frac{1}{a}\left[u(t)+(a-1)e^{-at}\right]$

Next: a beguilingly simple PDE

$$\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial t} = 0$$

Subject to boundary conditions: $w(x,0)=0;$ $w(0,t)=t$

Applying separation of variables, it is easy to show that

$$w(x,t) = e^{k\left(t-\frac{x^2}{2}\right)}$$

Apply the boundary conditions :

$w(x,0) = e^{-kx^2} = 0,$ all x!!

$w(0,t) = e^{kt} = 0,$ all t!!

In fact any function of the form $f\left(t-\frac{x^2}{2}\right)$, is a solution, eg $w(x,t) = t - \frac{x^2}{2}$

But runs into problems with boundary conditions

Try again, this time applying the LT

Again, consider the simple first order PDE: $$\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial t} = 0$$

Subject to boundary conditions: $w(x,0)=0; \qquad w(0,t)=t$

Of course, $w(x,t)$ is a function of x and t; but when we compute partial derivative of w with respect to t we treat x as a constant

First, the LT is a linear operator: $$L\left(\frac{\partial w}{\partial x}\right) + xL\left(\frac{\partial w}{\partial t}\right) = 0$$

The **second** term is easy: $$L\left(\frac{\partial w}{\partial t}\right) = sW - w(x,0) = sW$$

… but what about the first term?

Back to basics ...

$$L\left(\frac{\partial w}{\partial x}\right) = \int_0^\infty \frac{\partial w}{\partial x} e^{-st}\,dt = \frac{\partial}{\partial x}\int_0^\infty w(x,t)e^{-st}\,dt = \frac{\partial W}{\partial x}$$

Definition of LT

Integral of a derivative
= derivative of integral

Gathering the bits together ... $\dfrac{\partial W}{\partial x} + xsW = 0$

which we solve to get: $\displaystyle\int \frac{dW}{W} = -\int sx\,dx$

To find ... $W(x,s) = c(s)e^{-sx^2/2}$

Note that our separation of variables approach also gave the exponential term; but with a problematic constant – here we see that it is the ***Laplace variable*** s

Applying the final boundary condition: $w(0,t)=t$

Treating this as a function of t, we can take its LT $W(0,s)=\dfrac{1}{s^2}$

And, we found … $W(x,s)=c(s)e^{-sx^2/2}$

Evidently, $c(s)=\dfrac{1}{s^2}$

Consulting HLT or Kreysig p296 line 11, we find finally $w(x,t)=\left(t-\dfrac{1}{2}x^2\right)u\left(t-\dfrac{1}{2}x^2\right)$

Note the general form is still a function of $t-\dfrac{x^2}{2}$

$\boxed{\text{But the inclusion of the discontinuous step resolves our earlier difficulties with boundary conditions}}$

Back to waggling the semi-infinite string

Find the displacement *w(x,t)* of an elastic string subject to the following conditions:

1. The semi-infinite string is initially at rest, on the *x*-axis
2. For some time *t>0* the left hand end of the string (*x=0*) is moved sinusoidally:

$$w(0,t) = \begin{cases} \sin t & \text{if } 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

3. Furthermore

$$\lim_{x \to \infty} w(x,t) = 0, \text{ for } t \ge 0$$

We will tackle this problem using the Laplace Transform

*Kreysig, 8th Edn, page 644

** just in this part of the notes, we use *w(x,t)* for the PDE, rather than *u(x,t)* because *u(t)* is conventionally associated with the step function

To solve the wave equation

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

subject to $w(0,t) = f(t)$ and $\lim_{x\to\infty} w(x,t) = 0$ (for $t \geq 0$)

initial conditions $w(x,0) = 0$,

$$\left.\frac{\partial w}{\partial t}\right|_{t=0} = 0$$

FIRST, we take the LT with respect to t:

$$s^2 W(s) - s w(x,0) - \left.\frac{\partial w}{\partial t}\right|_{t=0} = c^2 L\left(\frac{\partial^2 w}{\partial x^2}\right)$$

The initial conditions mean that the second and third terms drop out

Recall formula for LT

$$L\left(\frac{\partial^2 w}{\partial x^2}\right) = \int_0^\infty e^{-st}\frac{\partial^2 w}{\partial x^2}dt$$

Exchanging the order of integration and differentiation :

$$L\left(\frac{\partial^2 w}{\partial x^2}\right) = \frac{\partial^2}{\partial x^2}\int_0^\infty e^{-st}w(x,t)dt = \frac{\partial^2}{\partial x^2}L(w) = \frac{\partial^2 W}{\partial x^2}$$

It follows that :

$$s^2 W = c^2\frac{\partial^2 W}{\partial x^2}$$

$$\frac{\partial^2 W}{\partial x^2} - \frac{s^2}{c^2}W = 0$$

so

$$W(x,s) = A(s)e^{\frac{sx}{c}} + B(s)e^{-\frac{sx}{c}}$$

Apply the boundary condition

$$F(s) = L(f(t)) = W(0,s)$$

Exchanging integration & differentiation again :

and so $$\lim_{x\to\infty} W(x,s) = \lim_{x\to\infty}\int_0^\infty e^{-st} w(x,t)\,dt = \int_0^\infty e^{-st} \lim_{x\to\infty} w(x,t)\,dt = 0$$

$$A(s) = 0$$
$$W(0,s) = B(s) = F(s)$$
$$W(x,s) = F(s)e^{-sx/c}$$

From HLT or from Kreysig page 296 (line 11), we have :

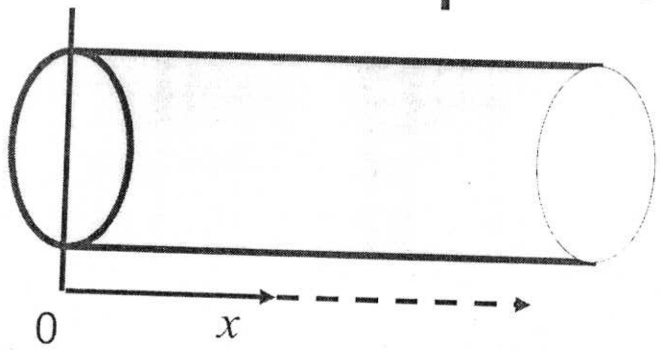
$$L\left(f\left(t-\frac{x}{c}\right)u\left(t-\frac{x}{c}\right)\right) = F(s)e^{-as}$$ (second shifting theorem)

and so $$w(x,t) = f\left(t-\frac{x}{c}\right)u\left(t-\frac{x}{c}\right)$$

that is :

$$\boxed{w(x,t) = \begin{cases} \sin\left(t-\frac{x}{c}\right) & \text{if } \frac{x}{c} < t < \frac{x}{c} + 2\pi \\ 0 & \text{otherwise} \end{cases}}$$

Heat equation example using Laplace Transform



We consider a semi-infinite insulated bar which is initially at a constant temperature, then the end $x=0$ is held at zero temperature.

We are to solve the Diffusion Equation: $\dfrac{\partial w}{\partial t} = c^2 \dfrac{\partial^2 w}{\partial x^2}$

Subject to the initial and boundary conditions:

$$w(x,0) = T_0$$

$$w(0,t) = 0$$

$$w(x,t) \to 0 \text{ as } x \to \infty$$

Applying the Laplace Transform

$$c^2 L\left(\frac{\partial^2 w}{\partial x^2}\right) = L\left(\frac{\partial w}{\partial t}\right)$$
$$= sW - w(x,0)$$
$$= sW - T_0$$

As usual: $$L\left(\frac{\partial^2 w}{\partial x^2}\right) = \frac{\partial^2 W}{\partial x^2}$$

and so $$c^2 \frac{\partial^2 W}{\partial x^2} = sW - T_0 \Rightarrow c^2 \frac{\partial^2 W}{\partial x^2} - sW = -T_0$$

We have CF : $W(x,s) = A(s)e^{-\frac{\sqrt{s}}{c}x} + B(s)e^{\frac{\sqrt{s}}{c}x}$

PI : $W(x,s) = \frac{T_0}{s}$

Solution is $W(x,s) = A(s)e^{-\frac{\sqrt{s}}{c}x} + B(s)e^{\frac{\sqrt{s}}{c}x} + \frac{T_0}{s}$

Evidently, *B(s)=0* from the third condition

UNIVERSITY QUESTION PAPERS

13RT/AD196.

Code No: 111AB

R13

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B.Tech I Year Examinations, June - 2014

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

1.a) Define an orthogonal matrix. [2m]

b) When a quadratic form is said to be [3m]
i) Positive definite ii) Negative definite iii) Positive semi definite.

c) State Rolle's Theorem. [2m]

d) When a function f(x, y), with usual notations of partial differential coefficients, will have maximum, minimum and can't be decided? [3m]

e) In evaluating $\iint_R f(x, y)dxdy$ bounded by the coordinate axes and the line $\frac{x}{a}+\frac{y}{b}=1$, find the limits of x and y. [2m]

f) Find the limits of integration after changing the order of integration of $\int_0^1 \int_{x^2}^{2-x} xydydx$. [3m]

g) State Law of Natural Growth. [2m]

h) Solve the differential equation $(D^2-3D+4)y=0$. [3m]

i) If $L[f(t)]=\frac{1}{(s-1)^2}$, then find $L^{-1}\left[\frac{1}{s(s-1)^2}\right]$ using any theorem of Laplace transforms. [2m]

j) Find $L(5\sin t+2\sin 3t)$. [3m]

PART-B

2. Using Cayley Hamilton theorem find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

OR

3. Find the Eigen values and the corresponding Eigen vectors of the matrix
$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

4. Expand $e^x \sin y$ in powers of x and y.

OR

5. Find the Maximum or minimum values of $f = 3x^4 - 2x^3 - 6x^2 + 6x + 1$.

6.a) Evaluate $\iint_R r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.

b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dxdydz$

OR

7.a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dxdy$

b) Evaluate $\iiint_V (xy + yz + zx) dxdydz$, where V is the region of space bounded by planes by $x = 0$, $x = 1$, $y = 0$, $y = 2$ and $z = 0$, $z = 3$.

8. If a voltage of 20 cos5t is applied to a series circuit consisting of 10 ohm resistor and 2 henry inductor, determine the current at any time t.

OR

9.a) Solve the differential equation $(D^2 - 2D + 1)y = x^2 e^{3x} - \sin 2x + 3$.

b) Bacteria in a culture grow exponentially so that the initial number has doubled in 3 hours. How many times, the initial number will be present after 9 hours.

10. Using Laplace transform solve the differential equation $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$, given that $x(0) = 2$, $x'(0) = -1$.

OR

11.a) Find the inverse Laplace transform of $\log\left(1 + \frac{16}{s^2}\right)$.

b) Find the Laplace transform of $f(t)$ where $f(t) = \begin{cases} t, & 0 < t < \frac{1}{2} \\ t-1, & \frac{1}{2} < t < 1 \\ 0, & t > 1 \end{cases}$.

****

Code No: Z0125 **R07**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, November/December - 2015

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, MCT, ETM, MMT, AE, BT)

Time: 3 hours **Max. Marks: 80**

Answer any five questions
All questions carry equal marks

\- - -

1.a) Solve $(y+x)dx=(y-x)dx$..

b) Obtain the orthogonal trajectories of the semi-cubical parabolas $ay^2=x^3$. [8+8]

2. Solve by the method of variation of parameters $(D^2-2D)y=e^x\sin x$. [16]

3.a) Find the minimum and maximum values of $\sin x+\sin y+\sin(x+y)$.

b) Verify Rolle's Theorem for $f(x)=x^3-4x+5$ in (1, 2). [8+8]

4.a) Trace the curve $r=a(1+\sin\theta)$.

b) Determine the centre of curvature to the curve in parametric form $x=3t^2,\ y=3t-t^3$. [8+8]

5.a) Find the volume generated by the revolution of the curve $r=2a\cos\theta$ about the initial line.

b) Change the order of integration and solve $\int_0^C\int_{x^2/a}^{2a-x} xy^2dydx$. [8+8]

6. Show that the given exponential series $1+\frac{x}{1!}+\frac{x^2}{1!}+\frac{x^3}{1!}+.....$ converges absolutely for all x. [16]

7. If $\vec{f}=3x^2yz^2\,i+x^2z^2\,j+2x^3yzk$. Show that $\int_C\vec{f}.d\vec{r}$ is independent of the path of integration. Hence evaluate the integral when C is any path joining (0, 0, 0) to (1, 2, 3). [16]

8.a) State and prove convolution theorem for Laplace transforms.

b) Find the Laplace transform of $L\left[e^{2t}\sin 3t\right]$. [8+8]

---ooOoo---

INTERNAL QUESTION PAPER WITH KEY

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

Narayanaguda, Hyderabad.

I B.Tech/IISEM. I Mid-Term Examinations, March – 2017

Mathematics II

Date: 6/3/17 Time: 1 Hour

Answer any two of the following

1. (a) Find the Laplace transform of $e^{-t}\cos 2t$. $\frac{s+1}{s^2+2s+5}$ CO1 Level 2

   (b) Find the inverse Laplace transform of $\frac{2s+12}{s^2+6s+13}$ $\frac{e^{-3t}}{4}(4\cos 2t+3\sin 2t)$ CO1, L2

2. (a) Find the Laplace transform of $\frac{1-\cos at}{t}$ $\log\sqrt{\frac{s^2+a^2}{s^2}}$ CO1, Level 2

   (b) Solve the initial value problem by using Laplace transform method CO1, L-2

   $(D^2+7D+10)y = 4e^{-3t}, y(0) = 0 \text{ and } y^1(0) = 0.$ $4\left[\frac{1}{6}e^{-5t}+\frac{1}{3}e^{-2t}-\frac{1}{2}e^{-3t}\right]$ C 3

3. (a) Show that $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$ CO2, L2

   (b) Show that $\int_0^1 x^n(\log x)^m\,dx = \frac{(-1)^m\,m!}{(n+1)^{m+1}}$ where n is positive integer, $n > -1$. 2 CO2, L2

4. (a) Evaluate $\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$ $\frac{\pi}{16}$ 3 CO3, L2

   (b) Evaluate $\iint r^3\,dr\,d\theta$ by taking $r = 2\sin\theta$ to $r = 4\sin\theta$ and $\theta = 0$ to $\theta = \pi$ $\frac{45\pi}{2}$ 2

   CO3, L2

Code No:132AB Set No. 4

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

I B.Tech. II Sem., I Mid-Term Examinations, March 2017

MATHEMATICS-II

Objective Exam

Name: ______________________ Hall Ticket No. [| | | | | A | | | |]

Answer All Questions. All Questions Carry Equal Marks. Time: 20 Min. Marks: 10.

I Choose the correct alternative:

1. $\Gamma(n)$ =-- [a] 1

(a) $(n-1)\Gamma(n-2)$ (b) $(n)\Gamma(n-1)$ (c) $(n-2)\Gamma(n-1)$ (d) $(n-1)\Gamma(n-1)$

2. Evaluate $\int_0^1 x^5(1-x)^3 dx$ =------------------------------- [a] 2

(a) $\frac{1}{504}$ (b) $\frac{2}{504}$ (c) $\frac{1}{50}$ (d) $\frac{1}{54}$

3. $\int_0^2\int_0^x y\,dx\,dy$ =-------------------------- [b] 3

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 4 (d) 3

4. $L\{f(t)\}$ = ------------ [a] 4

(a) $\int_0^\infty e^{-st} f(t)dt$ (b) $\int_0^\infty e^{st} f(t)dt$ (c) $\int_0^\infty e^{-st} f(s)dt$ (d) $\int_\infty^0 e^{-st} f(t)dt$

4. $L\{k\}$ =-- [b] 5

(a) k (b) $\frac{k}{s}$ (c) $\frac{1}{s}$ (d) s

6 $L\{u(t-a)\}$=-- [a] 6

(a) $\frac{e^{-at}}{s}$ (b) $\frac{e^{-as}}{a}$ (c) $\frac{e^{-as}}{s}$ (d) $\frac{e^{-at}}{t}$

7 $L^{-1}\left(\frac{1}{s^3}\right)$=-- [d] 7

(a) $\frac{t^{n-1}}{(n)!}$ (b) $\frac{t}{(n-1)!}$ (c) $\frac{t^{n-1}}{(n-2)!}$ (d) $\frac{t^{n-1}}{(n-1)!}$

8 $L^{-1}\left(\frac{4}{(s+1)(s+2)}\right)$ [a] 8

(a) $4(e^{-t}-e^{-2t})$ (b) $4(e^{-t}-e^{2t})$ (c) $4(e^{t}-e^{-2t})$ (d) $4(e^{-t}+e^{-2t})$

Cont....2

9 $\beta(m,n)$ =---------------------------- [b] 9

(a) $2\int_0^{\pi} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (b) $2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (c) $\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (d) none

10. Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^5}}$ in terms of Betafunction [c] 10

(a) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{5}\right)$ (b) $\beta\left(\frac{3}{5},\frac{1}{2}\right)$

(c) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right)$ (d) $\frac{1}{5}\beta\left(\frac{3}{2},\frac{1}{2}\right)$

II FIIL IN THE BLANKS

11. Evaluate $\int_0^{\infty} x^6 e^{-2x} dx$ ---- $\frac{\boxed{7}}{2^7}$ = ----------

12. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dx\, dy$ ---- $\pi/4$.

13. Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\, dr\, d\theta$ ---- $\frac{a^2\pi}{4}$.

14. Find $L(5\sin t + 2\sin 3t)$ ---- $\frac{5}{s^2+1} + \frac{6}{s^2+9}$.

15. Evaluate $L(te^t)$ ---- $\frac{1}{(s-1)^2}$

16. Find $L^{-1}\left\{\frac{1}{(s+2)^2+16}\right\}$ ---- $\frac{e^{-2t}\sin 4t}{4}$.

17. If $L^{-1}\{\bar{f}(s)\} = f(t)$, then $L^{-1}\{\bar{f}^{(n)}(s)\}$= ---- $(-1)^n t^n \cancel{\frac{d}{dt} f(t)}$

18. Beta function is defined as ---- $\int_0^{\infty} x^{m-1}(1-x)^{n-1} dx$.

19. Compute $\Gamma\left(\frac{11}{2}\right)$ = $\frac{945}{32}\sqrt{\pi}$

20. Write relation between Beta and Gamma function ---- $\beta(m,n) = \frac{\boxed{m}\,\boxed{n}}{\boxed{m+n}}$

$2x = y$.

$dx = \frac{dy}{2}$.

$\frac{1}{2}\left(\frac{y}{2}\right)^6 \cdot e^{-y} dy$.

$n-1 = 6$

$n = 7$.

$\frac{\boxed{7}}{2^7}$ //

M2 Mid I - Solutions

1) (a) find the laplace transform of $e^{-t}\cos 2t$.

Sol/

$$L[\cos 2t] = \frac{s}{s^2+4}$$

applying first shifting theorem, we get

$$L[e^{-t}\cos 2t] = [L\{\cos 2t\}] \text{ change } s \text{ to } s+1$$

$$= \left[\frac{s}{s^2+4}\right] \text{changes to } s+1$$

$$= \frac{s+1}{(s+1)^2+4} = \frac{s+1}{s^2+2s+5}$$

(b) Find the inverse laplace transform of

$$\frac{2s+12}{s^2+6s+13}$$

Sol/ let $F(s) = \dfrac{2s+12}{s^2+6s+13}$

$$= \frac{2(s+3)+3}{(s+3)^2+4}$$

$$L^{-1}[F(s)] = L^{-1}\left[\frac{2(s+3)+3}{(s+3)^2+2^2}\right]$$

$$= e^{-3t}L^{-1}\left[\frac{2s+3}{s^2+2^2}\right]$$, by first shifting theorem

$$= e^{-3t}\left[2\cdot L^{-1}\left\{\frac{s}{s^2+2^2}\right\} + 3\cdot L^{-1}\left\{\frac{1}{s^2+2^2}\right\}\right]$$

$$= e^{-3t}\left(2\cos 2t + 3\cdot\frac{1}{2}\sin 2t\right)$$

$$= \frac{e^{-3t}}{4}\left(4\cos 2t + 3\sin 2t\right).$$

2: (a) Find the laplace transform of $\frac{1-\cos at}{t}$

$$\therefore L\left\{\frac{1-\cos at}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+a^2}\right) ds$$

$$= \left[\log s - \frac{1}{2}\log(s^2+a^2)\right]_s^\infty$$

$$= \frac{1}{2}\left[2\log s - \log(s^2+a^2)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2}{s^2+a^2}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1}{1+\frac{a^2}{s^2}}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log 1 - \log\frac{s^2}{s^2+a^2}\right]$$

$$= -\frac{1}{2}\log\left(\frac{s^2}{s^2+a^2}\right)$$

$$= \log\left(\frac{s^2}{s^2+a^2}\right)^{-1/2}$$

$$= \log\sqrt{\frac{s^2+a^2}{s^2}}$$

2b) Solve the initial value problem by using Laplace transform method $(D^2+7D+10)y = 4e^{-3t}$

$y(0)=0$ and $y'(0) = -1$.

Sol/4 Given equation is

$$(D^2+7D+10)y = 4e^{-3t}$$

$$y''+7y'+10y = 4e^{-3t}$$

Taking laplace transform of both sides, we get

$$L[y''] + 7\cdot L[y'] + 10\cdot L[y] = 4\cdot L\{e^{-3t}\}.$$

i.e $$\left[s^2 L\{y\} - s\cdot y(0) - y'(0)\right] + 7\left[sL\{y\} - y(0)\right] + 10\,L\{y\} = 4\cdot\frac{1}{s+3}$$

Using the given conditions, it reduces to

$$\left[s^2 L\{y\} + 1\right] + 7sL\{y\} + 10L\{y\} = \frac{4}{s+3}$$

$$(s^2+7s+10)\,L\{y\} = \frac{4}{s+3} - 1 = \frac{4-s-3}{s+3} = \frac{1-s}{s+3}$$

$$L\{y\} = \frac{1-s}{(s+3)(s^2+7s+10)} = \frac{1-s}{(s+2)(s+3)(s+5)}$$

$$\therefore y = L^{-1}\left[\frac{1-s}{(s+2)(s+3)(s+5)}\right].$$

Consider $\frac{1-s}{(s+2)(s+3)(s+5)}$

Let $$\frac{1-s}{(s+2)(s+3)(s+5)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+5}$$

$$\Rightarrow 1-s = A(s+3)(s+5) + B(s+2)(s+5) + C(s+2)$$

(2) b Solve the initial value problem by using laplace transform method

$$(D^2+7D+10)y = 4e^{-3t}$$

$$y(0)=0 \text{ and } y'(0)=0.$$

soln

$$y''+7y'+10y = 4e^{-3t}$$

Taking laplace transform on both sides, we get

$$L[y''] + 7L[y'] + 10L[y] = 4 \cdot L[e^{-3t}].$$

i.e $\left[s^2 L\{y\} - s \cdot y(0) - y'(0)\right]$

$$+ \; 7\left[sL\{y\} - y(0)\right] + 10L\{y\} = \frac{4}{s+3}.$$

$$y(0)=0, \; y'(0)=0.$$

$$s^2L\{y\} + 7sL\{y\} + 10L\{y\} = \frac{4}{s+3}.$$

$$L\{y\} = \frac{4}{(s+3)(s^2+7s+10)}$$

$$L\{y\} = \frac{4}{(s+3)(s+2)(s+5)}$$

$$y = L^{-1}\left[\frac{4}{(s+3)(s+2)(s+5)}\right].$$

3 (a)

To show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$

proof :-

we have $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$ —(1)

Put $x = \frac{1}{1+y}$ so that $dx = \frac{-dy}{(1+y)^2}$

from (1) we have;

$$B(m,n) = \int_\infty^0 \frac{1}{(1+y)^{m-1}}\left(1 - \frac{1}{1+y}\right)^{n-1} \cdot \frac{-dy}{(1+y)^2}$$

$$= \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+1}(1+y)^{n-1}} dy = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$$

$$B(m,n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$$

Again since Beta function is symmetrical in m and n;

$$B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

$$\beta(m,n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

or $\beta(m,n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}} dy.$

3 b) Show that $\int_0^1 x^n (\log x)^m dx = \frac{(-1)^m m!}{(1+n)^{m+1}} dx$

Where 'n' is +ve integer. $n > -1$ (or) $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is +ve and $m > -1$

Sol: Put $\log x = -t$ i.e, $x = e^{-t}$ so that $dx = -e^{-t} dt$

Also when $x = 0$, $t = \infty$ and when $x = 1$, $t = 0$

$$\int_0^1 x^m (\log x)^n dx = \int_\infty^0 (e^{-t})^m (-t)^n (-e^{-t} dt)$$

$$= (-1)^n \int_0^\infty e^{-(m+1)t} t^n dt$$

$$= (-1)^n \int_0^\infty e^{-(m+1)t} t^{(n+1)-1} dt$$

$$= (-1)^n \frac{\Gamma(n+1)}{(m+1)^{n+1}} \quad \left[\because \int_0^\infty e^{-kx} x^{n-1} dx = \frac{\Gamma(n)}{k^n} \quad n > 0, k > 0 \right]$$

$$= \frac{(-1)^n n!}{(m+1)^{n+1}}$$

NOTE: $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}$

4.a) $\int_0^{\pi/2} \sin^2\theta \cos^2\theta \, d\theta$

we have $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2}\beta(m,n)$

$2m-1 = 2$ $\qquad$ $2n-1 = 2$

$2m = 3$ $\qquad$ $n = 3/2$

$m = 3/2$

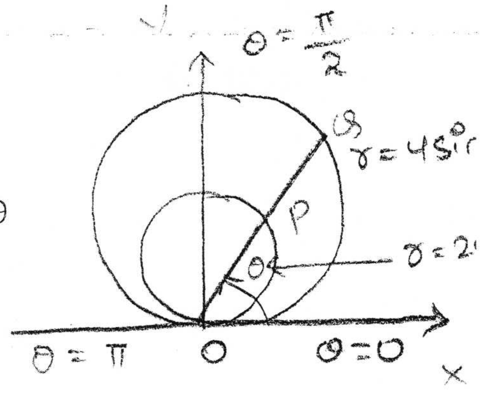
$$\int_0^{\pi/2} \sin^2\theta \cos^2\theta \, d\theta = \frac{1}{2}\beta\left(\frac{3}{2}, \frac{3}{2}\right)$$

$$= \frac{\frac{1}{2}\Gamma\frac{3}{2}\Gamma\frac{3}{2}}{\Gamma\left(\frac{3}{2}+\frac{3}{2}\right)}$$

$$= \frac{\frac{1}{2}\,\frac{1}{2}\Gamma\frac{1}{2}\,\frac{1}{2}\Gamma\frac{1}{2}}{\Gamma 3}$$

$$= \frac{\frac{1}{8}\pi}{\Gamma 2+1} = \frac{\frac{\pi}{8}}{2!} = \frac{\frac{\pi}{8}}{2} = \frac{\pi}{16}.$$

4b) Evaluate $\iint r^3\,dr\,d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.



$$\iint r^3\,dr\,d\theta = \int_{\theta=0}^{\pi}\int_{r=2\sin\theta}^{4\sin\theta} r^3\,dr\,d\theta$$

$$= \int_0^{\pi}\left\{\int_{2\sin\theta}^{4\sin\theta} r^3\,dr\right\}d\theta$$

$$= \int_0^{\pi}\left\{\frac{r^4}{4}\right\}_{2\sin\theta}^{4\sin\theta} d\theta$$

$$= \frac{1}{4}\int_0^{\pi}((256\sin^4\theta - 16\sin^4\theta)\,d\theta$$

$$= 60\int_0^{\pi}\sin^4\theta\,d\theta$$

$$= 60 \times 2\int_0^{\pi/2}\sin^4\theta\,d\theta$$

$$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \text{ if } f(2a-x) = f(x)\right.$$

$$= 120 \times \frac{3\times 1}{4\times 2}\,\frac{\pi}{2}$$

$$= \frac{45\pi}{2}$$

Code No:132AB Set No. 4

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

I B.Tech. II Sem., I Mid-Term Examinations, March 2017

MATHEMATICS-II

Objective Exam

Name: Hall Ticket No. | | | | | | A | | | | |

Answer All Questions. All Questions Carry Equal Marks. Time: 20 Min. Marks: 10.

I **Choose the correct alternative:**

1. $\Gamma(n)$ -- [A]

(a) $(n-1)\Gamma(n-2)$ (b) $(n)\Gamma(n-1)$ (c) $(n-2)\Gamma(n-1)$ (d) $(n-1)\Gamma(n-1)$

2. Evaluate $\int_0^1 x^{[illegible]}(1-x)^{[illegible]}dx$ -- [C]

(a) $\frac{1}{504}$ (b) $\frac{2}{504}$ (c) $\frac{1}{50}$ (d) $\frac{1}{51}$

3. $\int_0^{[illegible]}\int_0^{[illegible]} y\,dx\,dy$ ---------------------------- [b]

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 4 (d) 3

4. $L\{f(t)\}$ [ ]

(a) $\int_0^{\infty} e^{-st} f(t)dt$ (b) $\int_0^{\infty} e^{st} f(t)dt$ (c) $\int_0^{\infty} e^{-st} f(s)dt$ (d) $\int e^{-st} f(t)dt$

5. $L\{k\}$ -- [b]

(a) k (b) $\frac{k}{s}$ (c) $\frac{1}{s}$ (d) s

6. $L\{u(t-a)\}$ -- [C]

(a) $\frac{e^{-at}}{s}$ (b) $\frac{e^{-as}}{a}$ (c) $\frac{e^{-as}}{s}$ (d) $\frac{e^{-at}}{t}$

7. $L^{-1}\left(\frac{1}{s^n}\right)$ -- [d]

(a) $\frac{t^{n-1}}{(n)!}$ (b) $\frac{t}{(n-1)!}$ (c) $\frac{t^{n-1}}{(n-2)!}$ (d) $\frac{t^{n-1}}{(n-1)!}$

8. $L^{-1}\left(\frac{4}{(s+1)(s+2)}\right)$ [c]

(a) $4(e^{-t}-e^{-2t})$ (b) $4(e^{-t}-e^{2t})$ (c) $4(e^{t}-e^{-t})$ (d) $4(e^{-t}+e^{-2t})$

Cont....2

9 $\beta(m,n)$ ---------------------------------- [b]

(a) $2\int_0^{\pi} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (b) $2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (c) $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (d) none

10. Evaluate $\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^5}}$ in terms of Betafunction [c]

(a) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{5}\right)$ (b) $\beta\left(\frac{3}{5},\frac{1}{2}\right)$

(c) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right)$ (d) $\frac{1}{5}\beta\left(\frac{3}{2},\frac{1}{2}\right)$

II FIIL IN THE BLANKS

11. Evaluate $\int_0^{\infty} x^6 e^{-2x}\,dx$ $\frac{\Gamma 7}{2^7}$ ------------------ ------------

12. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ $\pi/4$ ------------------------------

13. Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$ $\frac{a^2\pi}{4}$ ------------------------------

14. Find $L(5\sin t + 2\sin 3t)$ $\frac{5}{s^2+1} + \frac{6}{s^2+9}$ ------------------------------

15. Evaluate $L(te^t)$ $\frac{1}{(s-1)^2}$ ------------------------------

16. Find $L^{-1}\left\{\frac{1}{(s+2)^2+16}\right\}$ $\frac{e^{-2t}\sin 4t}{4}$ ------------------------------

17. If $L^{-1}\{f(s)\} = f(t)$, then $L^{-1}\{f^{(n)}(s)\} =$ $(-1)^n t^n \frac{d^n f(t)}{dt^n}$ ------------------

18. Beta function is defined as $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ ------------------------------

19. Compute $\Gamma\left(\frac{11}{2}\right)$ $\frac{945}{32}\sqrt{\pi}$

20. Write relation between Beta and Gamma function $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ ------------------

Code No:132AB Set No. 1

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

I B.Tech. II Sem., I Mid-Term Examinations, March- 2017

Mathematics- II

Objective Exam

42

Name: Riddhi.M Hall Ticket No. 16BD1A1202

Answer All Questions. All Questions Carry Equal Marks. Time: 20 Min. Marks: 10.

I Choose the correct alternative:

1. $L\{f(t)\} =$ ------------ [a]

(a) $\int_0^\infty e^{-st} f(t)dt$ (b) $\int_0^\infty e^{st} f(t)dt$ (c) $\int_0^\infty e^{-st} f(s)dt$ (d) $\int_\infty^0 e^{-st} f(t)dt$

2. $L\{k\} =$ -- [b]

(a) k (b) $\frac{k}{s}$ (c) $\frac{1}{s}$ (d) s

3. $L\{u(t-a)\} =$ -- [c]

(a) $\frac{e^{-at}}{s}$ (b) $\frac{e^{-as}}{a}$ (c) $\frac{e^{-as}}{s}$ (d) $\frac{e^{-at}}{t}$

4. $L^{-1}\left(\frac{1}{s^3}\right) =$ -- [d]

(a) $\frac{t^{n-1}}{(n)!}$ (b) $\frac{t}{(n-1)!}$ (c) $\frac{t^{n-1}}{(n-2)!}$ (d) $\frac{t^{n-1}}{(n-1)!}$

5. $L^{-1}\left(\frac{4}{(s+1)(s+2)}\right)$ [a]

(a) $4(e^{-t} - e^{-2t})$ (b) $4(e^{-t} - e^{2t})$ (c) $4(e^{t} - e^{-2t})$ (d) $4(e^{-t} + e^{-2t})$

6. $\beta(m,n) =$ ---------------------------- [c]

(a) $2\int_0^\pi \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$ (b) $2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$

(c) $\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$ (d) none

Cont 2....

7. Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^5}}$ in terms of Betafunction [a]

(a) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{5}\right)$ (b) $\beta\left(\frac{3}{5},\frac{1}{2}\right)$ (c) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right)$ (d) $\frac{1}{5}\beta\left(\frac{3}{2},\frac{1}{2}\right)$

8. $\Gamma(n)$ = -- [b]

(a) $(n-1)\Gamma(n-2)$ (b) $(n)\Gamma(n-1)$ (c) $(n-2)\Gamma(n-1)$ (d) $(n-1)\Gamma(n-1)$

9. Evaluate $\int_0^1 x^5(1-x)^3 dx$ = ------------------------------------ [a]

(a) $\frac{1}{504}$ (b) $\frac{2}{504}$ (c) $\frac{1}{50}$ (d) $\frac{1}{54}$

10. $\int_0^2\int_0^x y\,dx\,dy$ = -------------------------------- [b]

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 4 (d) 3

II FIIL IN THE BLANKS

11. Find $L(5\sin t + 2\sin 3t)$ ---- $5/_{s^2+1} + 2/_{s^2+9}$ ----

12. Evaluate $L(te^t)$ --

13. Find $L^{-1}\left\{\frac{1}{(s+2)^2+16}\right\}$ ------------------------------------

14. If $L^{-1}\{\bar{f}(s)\} = f(t)$, then $L^{-1}\{\bar{f}^{(n)}(s)\} =$ --------------------

15. Beta function is defined as ----------------------------------

16. Compute $\Gamma\left(\frac{11}{2}\right)$ = $\frac{945}{32}$

17. Write relation between Beta and Gamma function - $\frac{\Gamma m \Gamma n}{\Gamma m+n}$

18. Evaluate $\int_0^\infty x^6 e^{-2x} dx$ ------------------ -----------

19. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} dx\,dy$ --------------------------

20. Evaluate $\int_0^\pi \int_0^{a\sin\theta} r\,dr\,d\theta$ -----------------------------

Code No:132AB Set No. 2

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

I B.Tech. II Sem., I Mid-Term Examinations, March 2017

MATHEMATICS-II

Objective Exam

Name: N. Divyaani Kaushal. Hall Ticket No. 16BD1A7231

Answer All Questions. All Questions Carry Equal Marks. Time: 20 Min. Marks: 10.

I Choose the correct alternative:

1. $L^{-1}\left(\frac{1}{s^3}\right)$ = -- [a]

(a) $\frac{t^{n-1}}{(n)!}$ (b) $\frac{t}{(n-1)!}$ (c) $\frac{t^{n-1}}{(n-2)!}$ (d) $\frac{t^{n-1}}{(n-1)!}$

2. $L^{-1}\left(\frac{4}{(s+1)(s+2)}\right)$ [a]

(a) $4(e^{-t} - e^{-2t})$ (b) $4(e^{-t} - e^{2t})$ (c) $4(e^{t} - e^{-2t})$ (d) $4(e^{-t} + e^{-2t})$

3. $\beta(m,n)$ = ---------------------------- [b]

(a) $2\int_0^{\pi} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (b) $2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$

(c) $\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (d) none

4. Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^5}}$ in terms of Betafunction [c]

(a) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{5}\right)$ (b) $\beta\left(\frac{3}{5},\frac{1}{2}\right)$ (c) $\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right)$ (d) $\frac{1}{5}\beta\left(\frac{3}{2},\frac{1}{2}\right)$

5. $\Gamma(n)$ = -- [a]

(a) $(n-1)\Gamma(n-2)$ (b) $(n)\Gamma(n-1)$ (c) $(n-2)\Gamma(n-1)$ (d) $(n-1)\Gamma(n-1)$

6. Evaluate $\int_0^1 x^5(1-x)^3 dx$ = ------------------------------------ [a]

(a) $\frac{1}{504}$ (b) $\frac{2}{504}$ (c) $\frac{1}{50}$ (d) $\frac{1}{54}$

7. $\int_0^2\int_0^x y\,dx\,dy$ = -------------------------- [b]

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 4 (d) 3

Cont 2....

8. $L\{f(t)\} =$ ------------ [a]

(a) $\int_0^\infty e^{-st} f(t)dt$ (b) $\int_0^\infty e^{st} f(t)dt$ (c) $\int_0^\infty e^{-st} f(s)dt$ (d) $\int_\infty^0 e^{-st} f(t)dt$

9. $L\{k\}$ = ------------------------------ [b]

(a) k (b) $\frac{k}{s}$ (c) $\frac{1}{s}$ (d) s

10. $L\{u(t-a)\}$ = ------------------------------ [c]

(a) $\frac{e^{-at}}{s}$ (b) $\frac{e^{-as}}{a}$ (c) $\frac{e^{-as}}{s}$ (d) $\frac{e^{-at}}{t}$

Code No:132BA **:2:** **Set No. 2**

II FIIL IN THE BLANKS

11. If $L^{-1}\{\bar{f}(s)\} = f(t)$, then $L^{-1}\{\bar{f}^{(n)}(s)\} =$ $\cancel{(-1)^n t^n f(t)}$ $(-1)^n t^n f(t)$.

12. Beta function is defined as $B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$.

13. Compute $\Gamma\left(\frac{11}{2}\right) = \frac{9}{2} \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\sqrt{\frac{1}{2}} = \frac{945\sqrt{\pi}}{32}$

14. Write relation between Beta and Gamma function $B(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$

15. Evaluate $\int_0^\infty x^6 e^{-2x} dx$ ---- $\frac{6!}{2^7}$ ----

16. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ ------------

17. Evaluate $\int_0^\pi \int_0^{a\sin\theta} r\, dr\, d\theta$ ---- $\frac{\pi a^2}{2}$ ----

18. Find $L(5\sin t + 2\sin 3t)$ ---- $\frac{5}{s^2+1} + \frac{6}{s^2+9} = \frac{5s^2+45+6s^2+6}{(s^2+1)(s^2+9)} = \frac{11s^2+51}{(s^2+1)(s^2+9)}$

19. Evaluate $L(te^t)$ ---- $\cancel{\frac{1}{s+1}}$ $\frac{1}{(s-1)^2}$ ----

20. Find $L^{-1}\left\{\frac{1}{(s+2)^2+16}\right\}$ ---- $e^{-2t}\frac{1}{4}\sin 4t$. ----

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY (KMIT)

3-5-1026, Narayanaguda, Hyderabad-29. Ph. 23261407

MID EXAMINATION : I / II / III

Name : P. Ruthvik. Roll No. : 1 6 B D 1 A 1 2 4 3

Course : B.Tech Subject :

Branch : Section : IT Date : 6/3/17

1. Lapalace transform

$e^{-t} \cos 2t$

$\Rightarrow \dfrac{2 - \cos at}{t}$

$\Rightarrow 1 - \cos at = t$

$\Rightarrow 1 - \cos at^2 = 0.$

$\Rightarrow \dfrac{1s + \cos at^2}{t} = 0.$

$\Rightarrow \dfrac{2s + 12}{s^2 + 6s + 13}$

$\Rightarrow \dfrac{1 - \cos at}{t}$

$\Rightarrow B(m,n)$

$\dfrac{(-1)^t \, at!}{(m+n).}$

$\Rightarrow n > -1$

Question No.	Marks	
	Max	Obtained
1a.		
1b.		
1c.		
2a.		
2b.		
2c.		
3a.		
3b.		
3c.		
4a.		
4b.		
4c.		
Total		
Signature		

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY (KMIT)

3-5-1026, Narayanaguda, Hyderabad-29. Ph. 23261407

MID EXAMINATION : I / II / III

Name : Pabboju Vaishnavi Roll No. : 1 6 B D 1 A 1 2 3 7

Course : B.Tech Sem-2 Mid-1 Subject : M2

Branch : IT Section : — Date : 6-3-17

10

1 Sol:- (a) given $e^{-t}\cos 2t$

$L\{e^{-t}\cos 2t\}$

Considering $L\{\cos 2t\}$

$\because L\{\cos at\} = \frac{s}{s^2+a^2}$

$\therefore L\{\cos 2t\} = \frac{s}{s^2+4}$

From first shifting theorem

$L\{e^{-at}f(t)\} = f(s+a)$

$\therefore L\{e^{-t}\cos 2t\} = \frac{(s+1)}{(s+1)^2+4}$

(b) given $L^{-1}\left\{\frac{2s+12}{s^2+6s+13}\right\}$

$\Rightarrow L^{-1}\left\{\frac{2s+12}{s^2+6s+9+4}\right\} = L^{-1}\left\{\frac{2s+12}{(s+3)^2+(2)^2}\right\}$

$\Rightarrow L^{-1}\left\{\frac{2(s+3)+6}{\ldots}\right\}$

Question No.	Marks	
	Max	Obtained
1a.		
1b.		
1c.		
2a.		
2b.		
2c.		
3a.		
3b.		
3c.		
4a.		
4b.		
4c.		
Total		
Signature		

$$\Rightarrow L^{-1}\left\{\frac{2(s+3)}{(s+3)^2+(2)^2}\right\} + L^{-1}\left\{\frac{6}{(s+3)^2+(2)^2}\right\}$$

Considerin $L^{-1}\left\{\frac{2(s+3)}{(s+3)^2+(2)^2}\right\}$

from $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$ & first shifting theorem

$$2\, L^{-1}\left\{\frac{(s+3)}{(s+3)^2+(2)^2}\right\} = 2e^{-3t}\cos 2t.$$

considering $L^{-1}\left\{\frac{6}{(s+3)^2+(2)^2}\right\}$

from $L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at$ & from first shifting theorum

$$3\, L^{-1}\left\{\frac{2}{(s+3)^2+(2)^2}\right\} = 3e^{-3t}\sin 2t.$$

$$\therefore\ L^{-1}\left\{\frac{2s+12}{s^2+6s+13}\right\} = 3e^{-3t}\sin 2t + 2e^{-3t}\cos 2t.$$

(a) given $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx.$

w.k.T $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

U.L $x = \frac{1}{1+y}$ L.L $x = \frac{1}{1+y}$.

$y = 0$ $y = \infty$

Sub the values

$$\Rightarrow \int_{\infty}^{0} \left(\frac{1}{1+y}\right)^{m-1} \left(1 - \frac{1}{1+y}\right)^{n-1} \frac{-dy}{(1+y)^2} = \beta(m,n)$$

$$\Rightarrow \int_{0}^{\infty} \frac{1}{(1+y)^{m-1}} \frac{(1+y-1)^{n-1}}{(1+y)^{n-1}} \frac{dy}{(1+y)^2} = \beta(m,n)$$

$$\Rightarrow \int_{0}^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy = \beta(m,n)$$

$\because$ $\beta(m,n) = \beta(n,m)$

replace $x = y$

$dx = dy$

$$\therefore \int_{0}^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx = \beta(m,n)$$

Proved.

(b) given $\int_{0}^{1} x^n (\log x)^m dx = \frac{(-1)^m m!}{(n+1)^{m+1}}$

put $\log x = -t$

$\Rightarrow$ $x = e^{-t}$ U.L

$dx = -e^{-t} dt$ $\log 1 = -t$

$t = 0$

$$\Rightarrow \int_{\infty}^{0} e^{-tn} (-t)^m (-e^{-t})\, dt$$

$$\Rightarrow \int_{0}^{\infty} e^{-(n+1)t} (-t)^m \, dt$$

Let us put $(n+1)t = y$

$$\Rightarrow dt = \frac{dy}{n+1}$$

$$\Rightarrow \int_{0}^{\infty} e^{-y} \left(\frac{-y}{(n+1)}\right)^m \frac{dy}{(n+1)}$$

$$\Rightarrow (-1)^m \int_{0}^{\infty} e^{-y} y^m \frac{dy}{(n+1)^{m+1}} = \frac{(-1)^m}{(n+1)^{m+1}} \int_{0}^{\infty} e^{-y} y^m \, dy$$

$$\because \int_{0}^{\infty} e^{-x} x^{n-1} \, dx = \Gamma n$$

$$= \frac{(-1)^m \Gamma m}{(n+1)^{m+1}}$$

$$\therefore \int_{0}^{\infty} x^n (\log x)^m \, dx = \frac{(-1)^m \Gamma m}{(n+1)^{m+1}}$$

(a) given $\frac{1-\cos at}{t}$

$$L\left\{\frac{1}{t} - \frac{\cos at}{t}\right\} = L\left\{\frac{1}{t}\right\} - L\left\{\frac{\cos at}{t}\right\}$$

consider $L\left\{\frac{1}{t}\right\}$

w.k.T $L\{1\} = \frac{1}{s}$

division by t $L\left\{\frac{1}{t}\right\} = \cancel{\int_{s}^{\infty}\left(\frac{1}{s}\right) dt}$

$$\int_{s}^{\infty}\left(\frac{1}{x}\right) dx = \log\infty - \log s$$

$$\therefore L\left\{\frac{1}{t}\right\} = -\log s.$$

consider $L\left\{\frac{\cos at}{t}\right\}$

w.k.T $L\{\cos at\} = \frac{s}{s^2+a^2}$

division by t $L\left\{\frac{\cos at}{t}\right\} = \int_{s}^{\infty} \frac{x}{x^2+a^2}\, dx$

$$\Rightarrow \frac{1}{2}\int_{s}^{\infty} \frac{2x}{x^2+a^2}\, dx = \frac{1}{2}\log(x^2+a^2)_{s}^{\infty}$$

$$= -\frac{1}{2}\log(s^2+a^2)$$

$$L\left\{\frac{1-\cos at}{t}\right\} = -\log s + \frac{1}{2}\log(s^2+a^2)$$

(b) $(D^2+7D+10)y = 4e^{-3t}$, $y(0)=0$ $y'(0)=0$.

$\cancel{L\{D^2+7D+}$

$$y''+7y'+10y = 4e^{-3t}$$

applying Laplace transformations on b.s

$$L\{y''+7y'+10y\} = \cancel{4e^{-3t}} \; L\{4e^{-3t}\}$$

$$L\{y''\} + 7L\{y'\} + 10L\{y\} = L\{4e^{-3t}\}$$

$$s^2L\{y\} - sy(0) - y'(0) + 7[sL\{y\} - y(0)] +$$

$$10L\{y\} = \frac{4}{s+3}$$

$$L\{y\}[s^2+7s+10] = \frac{4}{s+3}$$

$$L\{y\} = \frac{4}{(s+3)(s^2+7s+10)} = \frac{4}{(s+3)(s+2)(s+5)}$$

Consider $$\frac{4}{(s+3)(s+2)(s+5)} = \frac{A}{s+3} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$\Rightarrow \frac{4}{(s+3)(s+2)(s+5)} = \frac{-2}{s+3} + \frac{4}{3}\frac{1}{s+2} + \frac{2}{3}\frac{1}{s+5}$$

$$y = L^{-1}\left\{\frac{-2}{s+3}\right\} + L^{-1}\left\{\frac{4}{3(s+2)}\right\} + L^{-1}\left\{\frac{2}{3}\frac{1}{(s+5)}\right\}$$

$$= -2e^{-3t} + \frac{4}{3}e^{-2t} + \frac{2}{3}e^{-5t}$$

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

Narayanguda, Hyderabad

I - B.Tech – II – Semester – R16 – II – Mid Internal Examinations March - 2017

Sub: Mathematics - II | Date: 10-05-17 – AN

Branch / Section: ECE, CSE, EIE & IT | Duration: 90 Min | Max Marks: 10

Answer any two of the following

1 a) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$ $\quad \frac{\pi^2}{4}.$ 1M(CO3,LEVEL2)

b) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xydxdy$ 4M (CO3,LEVEL3)

$$\frac{1}{2}\int_0^1 y^2 dy + \frac{1}{2}\int_1^2 (4y-4y^2+y^3)dy = \frac{3}{8}$$

2 a) Find the area of the loop of curve $r=a(1+\cos\theta)$ $= 3\pi a^2/2$ 2.5M(CO3,LEVEL2)

b) Find the directional derivative of $f=x^2-y^2+2z^2$ at the point p=(1,2,3) in the direction of line PQ where Q=(5,0,4) 2.5M (CO4,LEVEL2)

$$\bar{e} = \frac{4i-2j+k}{\sqrt{21}}.$$

$$= \frac{28}{\sqrt{21}}$$

3 a) Find constants a,b,c so that the vector $\bar{f}$= (x+2y+az)i +(bx-3y-z)j+(4x+cy+2z)k is irrotatinal.

$\cancel{a=2}$ $\cancel{b=3}$, $\cancel{c=3}$ $\quad a=4, b=2, c=-1$ 2M(CO4,LEVEL2)

b) Find the circulation of $\bar{F}$=(2x-y+2z)i+(x+y-z)j+(3x-2y-5z)k along the cicle $x^2+y^2=4$ in the plane.

$=8\pi$

3M(CO5,LEVEL2)

4. Verify Guass divergence theorem for $\bar{F}=x^3i+y^3j+z^3k$ taken over the cube bounded by x=0,x=a, y=0,y=a,z=0,z=a. 5M(CO5,LEVEL3)

Ans : $3a^5$

Code No:MA202BS Set No. 4

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

I B.Tech. II-Sem., II Mid-Term Examinations, May – 2017

MATHEMATICS - II

Objective Exam

Name: ______________________ Hall Ticket No. | | | | | |A| | | | |

Answer All Questions. All Questions Carry Equal Marks. Time: 20 Min. Marks: 10.

I Choose the correct alternative:

1. If $\vec{a}$ and $\vec{b}$ are irrotational vectors, Then $\vec{a}\times\vec{b}$ is ---------------- [a]

   a) Solenoidal b) irrotaticonal c) free vector d) none

2. If $\phi = x^2 + y^2 + z^2 - 3xyz$ then curl $(\text{grad}\,\phi) =$ ----------------- [C]

   a) 6x + 6y + 6z b) x + y + z c) 0 d) None

3. The value of $\int_0^3 \int_0^2 (4-y)^2 dy\, dx$ [a]

   a) 16 b) $\frac{16}{3}$ c) $\frac{8}{3}$ d) none

4. $\iint \frac{xy}{\sqrt{1-y^2}} dx\, dy$ over the positive quadrant of $x^2 + y^2 = 1$ is ------------- [a]

   a) $\frac{1}{6}$ b) $\frac{2}{3}$ c) $\frac{5}{6}$ d) none

5. $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dxdydz$ = ------ [C]

   a) 12 b) 24 c) 48 d)36

6. $\int_0^{\pi}\int_0^{a\cos\theta} r\sin\theta\, drd\theta$ = ------------- [b]

   a) $\frac{a^2}{2}$ b) $\frac{a^2}{3}$ c) $\frac{a^3}{3}$ d) $\frac{a^3}{4}$

Cont...2

7. The limits of $\iint dxdy$ over the region bounded by $y = x^2$ and x are [a]

a) x=0 to 1 ; $y = \sqrt{x}$ *to* x^2 b) x=0 to 1; y=0 to 1 c) $x = y^2$; $y = 0$ *to* 1 d) none

8. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, along the line y=x from (0,0) to (1,2) on XY plane then $\oint_C \vec{r} \bullet dr$ [b]

a) 0 b) 1 c)2 d) 4

9. For any open surface S $\iint_S curl\ \vec{f}\ .\vec{n}\ ds$ = -------------- [d]

a) 0 b) π c)2 $\vec{f}$ d) $\oint_C \vec{f} \bullet dr$

10. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ Then curl $(\vec{r})$=-------- [b]

a) $3x\vec{i}$ b) $\bar{0}$ c) $3y\vec{j}$ d) $3z\vec{k}$

II FIIL IN THE BLANKS

11. Directional derivative of ϕ in the direction of $\vec{f}$ is $\nabla\phi \cdot \bar{e}$ where $\bar{e} = \frac{\bar{f}}{|\bar{f}|}$

12. Unit normal vector to the surface $z = x^2 + y^2$ at (-1,-2,5) is $\frac{2\hat{i} + 4\hat{j} + k}{\sqrt{21}}$

13. Angle between two surfaces ϕ_1 and ϕ_2 is $\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$

14. If div $\vec{f} = 0$ then $\vec{f}$ is Solenoidal

15. By Guss divergence $\int_S \vec{f} \bullet \vec{n}$ ds = $\int_V div\ \bar{f}\, dv$

16. By Green's theorem $\oint_C Mdx + Ndy$ = $\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

17. By Stokes them $\oint_c \vec{f} \bullet d\vec{r}$ = $\int_S curl\ \bar{F} \cdot \bar{n}\, ds$

18. $\int_0^2 \int_0^x (x + y) dxdy$ = 4

19. $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}}$ f(x,y) dy dx after changing the order of integration $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

20. $\nabla\phi$ = $\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$

1(a). $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$

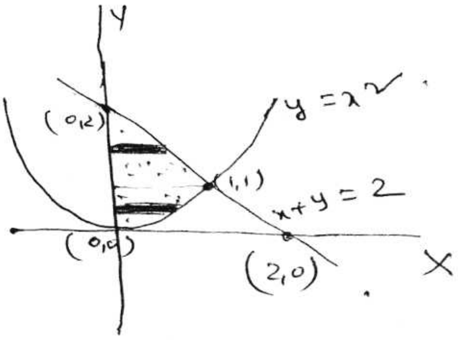
$= \int_0^1 \frac{dx}{\sqrt{1-x^2}} \times \int_0^1 \frac{dy}{\sqrt{1-y^2}}$

$= \left(\sin^{-1}x\right)_0^1 \left(\sin^{-1}y\right)_0^1$

$= \frac{\pi}{2}\cdot\frac{\pi}{2}$

$= \frac{\pi^2}{4}.$

1(b). $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy.$



$\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy = \int_0^1 \int_0^{\sqrt{y}} xy\,dx\,dy + \int_1^2 \int_0^{2-y} xy\,dx\,dy.$

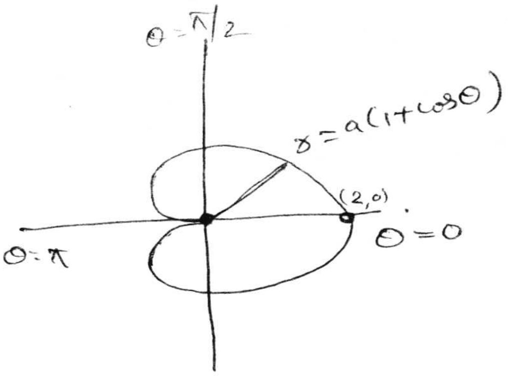
$= \frac{1}{2}\left[\left(\frac{y^3}{3}\right)_0^1 + \left[2y^2 + \frac{y^4}{4} - \frac{4y^3}{3}\right]_1^2\right]$

$= \frac{1}{2}\left[1 - \frac{1}{4}\right]$

$= \frac{3}{8}$

2(a)

W.K.T.



$$\text{area} = \iint_{r\ \theta} r\,dr\,d\theta$$

$$2\int_{r=0}^{a(1+\cos\theta)} \int_{\theta=0}^{\pi} r\,dr\,d\theta$$

$$= \cancel{2}\int_{\theta=0}^{\pi} \left(\frac{r^2}{\cancel{2}}\right)_0^{a(1+\cos\theta)} d\theta$$

$$= \int_0^{\pi} \left[a^2(1+\cos\theta)^2\right] d\theta$$

$$= \frac{3\pi a^2}{4}$$

2(b) $f = x^2 - y^2 + 2z^2$ ✓ $P(1,2,3)$ $Q = (5,0,4)$

$$D.D = \nabla f \cdot \frac{\overline{PQ}}{|\overline{PQ}|}$$

$$\nabla f = 2xi - 2yj + 4zk$$

$$(\nabla f)(1,2,3) = 2i - 4j + 12k$$

$$\overline{PQ} = 4i - 2j + k$$

$$\therefore D.D = (2i - 4j + 12k) \cdot \frac{(4i - 2j + k)}{\sqrt{21}}$$

$$= \frac{8+8+12}{\sqrt{21}} = \frac{28}{\sqrt{21}}$$ //

i) $\bar{A} = (x+2y+az)i + (bx-3y-z)j + (4x+cy+2z)K$.

Sol) w.k.T. $\bar{A}$ is irrotational if curl $\bar{A} = 0$,

$$\begin{vmatrix} i & j & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix} = 0,$$

$\Rightarrow$ $i[c+1] - j[4-a] + k[b-2] = 0$

$\Rightarrow$ $c = -1$, $a = 4$, $b = 2$.

(3b) $\bar{F} = (2x-y+2z)i + (x+y-z)j + (3x-2y-5z)K$,

& circle $x^2+y^2=4$, xy-plane,

circulation $= \int_C \bar{F}\cdot d\bar{r} = \int_C F_1 dx + F_2 dy + F_3 dz$

xy-plane $z = 0$, $dz = 0$,

$= \int_C (2x-y)\,dx + (x+y)\,dy + 0$,

$x = 2\cos\theta$, $y = 2\sin\theta$

$dx = -2\sin\theta\, d\theta$, $dy = 2\cos\theta\, d\theta$

θ: 0 to 2π,

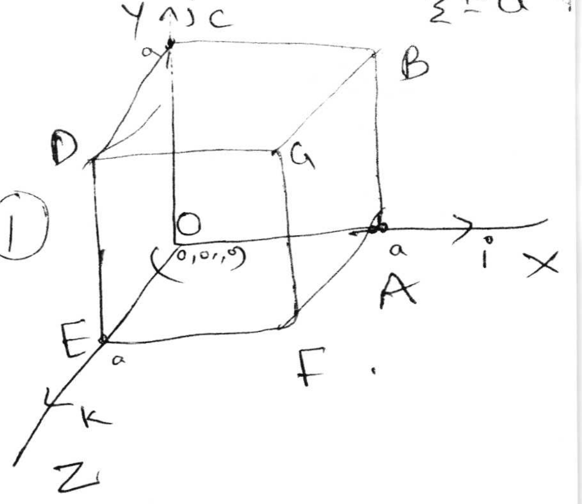
$= \int_{\theta=0}^{2\pi} (2(2\cos\theta) - 2\sin\theta)(-2\sin\theta)\,d\theta + (2\cos\theta + 2\sin\theta)\,2\cos\theta\, d\theta$

$= 8\pi$ //

4) $\bar{F} = x^3 i + y^3 j + z^3 K$, $x=0, x=a, z=0, y=a, z=a$ $z=a$

From Gauss Divergence thm

$$\boxed{\int_S \bar{F}\cdot\bar{n}\,ds = \int_V \operatorname{div}\bar{F}\,dV} \quad \text{--- (1)}$$



$\operatorname{div}\bar{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

$= 3x^2 + 3y^2 + 3z^2$

RHS $\int_V \operatorname{div}\bar{F}\,dv = 3\int_{x=0}^{a}\int_{y=0}^{a}\int_{z=0}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$

$= 3\int_{x=0}^{a} x^2 dx \times \int_{y=0}^{a} dy \times \int_{z=0}^{a} dz + 3\int_{x=0}^{a} dx \times \int_{y=0}^{a} y^2 dy \times \int_{z=0}^{a} dz$

$+ 3\int_{x=0}^{a} dx \times \int_{y=0}^{a} dy \times \int_{z=0}^{a} z^2 dz$

$= 3a^5$

LHS. $\int_S \bar{F}\cdot\bar{n}\,ds = \int_{S_1} \bar{F}\cdot\bar{n}\,ds + \int_{S_2} \bar{F}\cdot\bar{n}\,ds + \int_{S_3} \bar{F}\cdot\bar{n}\,ds + \int_{S_4} \bar{F}\cdot\bar{n}\,ds + \int_{S_5} \bar{F}\cdot\bar{n}\,ds$

$+ \int_{S_6} \bar{F}\cdot\bar{n}\,ds$,

Let S_1 : EFGD in xy-plane.

$\bar{n} = \bar{K}$ $ds = dx\,dy$

$z = a$. $\bar{F}\cdot\bar{n} = \bar{F}\cdot K = z^3 = a^3$

$\int \bar{F}\cdot\bar{n}\,ds = \int_0^a\int_0^a a^3\,dx\,dy = a^3 \cdot a \cdot a = a^5$

S_2 : OABC in xy-plane, $n = -k$, $z = 0$

$ds = dxdy$ $\bar{F} \cdot \bar{n} = \bar{F} \cdot (-k) = -z^3 = 0$,

$\therefore \int_{S_2} \bar{F} \cdot \bar{n} ds = 0$.

S_3 : surface in yz-plane. $\bar{n} = -\bar{i}$, $x = 0$

$\int_{S_3} \bar{F} \cdot \bar{n} ds = \int_0^a \int_0^a -x^3 dy dz = 0$,

$\int_{S_4} \bar{F} \cdot \bar{n} ds = a^5$

$\int_{S_5} \bar{F} \cdot \bar{n} ds = 0$

$\int_{S_6} \bar{F} \cdot \bar{n} ds = a^5$

$\therefore \int_S \bar{F} \cdot \bar{n} ds = a^5 + a^5 + a^5 = 3a^5$.

ASSIGNMENT QUESTION PAPERS

2016-17

MATHEMATICS – II

ASSIGNMENT-1

I YR - II SEM

1. (a) Find $L\left\{\int_0^t \frac{e^t\, sint}{t}\, dt\right\}$ (CO 1 LEVEL 2)

   (b) Find $L^{-1}\left\{\int_0^t \frac{s+3}{(s^2+6s+13)^2}\right\}$ (CO 1 LEVEL 2)

2. (a) Using convolution theorem find $L^{-1}\left\{\int_0^t \frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ (CO 1 LEVEL 3)

   (b) Using Laplace trance form, solve the differential equation (CO 1 LEVEL 2)

   $(D^2+2D+5)y = e^{-t} sint$ given that $y(0)=0,\ y^1(0)=1.$

3. (a) Prove that $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}\, dx.$ (CO 2 LEVEL 2)

   (b) Show that $\int_0^\infty x^n\, e^{-a^2x^2} dx = \frac{1}{2a^{n+1}}\Gamma\left(\frac{n+1}{2}\right)$ $n > -1$ and deduce that

   $\int_0^\infty cos(x^2)\, dx = \int_0^\infty sin(x^2)\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$ (CO 2 LEVEL 3)

   Hence evaluate $\int_{-\infty}^\infty e^{-a^2x^2} dx$.

4. (a) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}} dx = \frac{\pi}{4\sqrt{2}}$ (CO 2 LEVEL 2)

   (b) Prove that $B(m,n) = \frac{\Gamma m\, \Gamma n}{\Gamma(m+n)}$ (CO 2 LEVEL 2)

5. (a) Evaluate $\int_0^{\pi/2}\int_0^\infty \frac{r\, dr\, d\theta}{(r^2+a^2)^2}$ (CO 3 LEVEL 2)

   (b) Find the value of $\iint xy\, dx\, dy$ taken over the positive quadrant of the ellipse

   $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (CO 3 LEVEL 3)

ASSIGNMENT -2

M-2

1. Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dz\, dx\, dy$ CO 3 Level 3
2. Find the angle between the surface $x^2+y^2+z^2=9$ and $z= x^2+y^2-3$ at the point (2,-1,2). CO4 Level 2
3. Prove that grad$(\bar{a}.\bar{b})$ = $(\bar{b}.\nabla)\bar{a}$ + $(a.\nabla)\bar{b}$ +$\bar{b}\ X\ curl\bar{a}$ +$\bar{a}\ X\ curl\bar{b}$. CO4 Level 3
4. Evaluate $\int_s \bar{F}.\, N\, ds$ where $\bar{F} = 12x^2yi - 3yzj + 2zk$ and S is the portion of the plane x+y+z=1 included in the first octant. CO4 Level 2
5. Veryfy Greens theorem in plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the region bounded by $y=\sqrt{x}$ and $y= x^2$. CO5 Level 3
6. Veryfy Stoke theorem for the function $\bar{F} = x^2i + xyj$ integrated round thd square in the plane z=0 whose sides are a long the lines x=0,y=0,x=a,y=a. CO5 Level 3

5

N. SRIKAVYA REDDY

IT

16BD1A1232

M-2 ASSIGNMENT-2

SEM - II

1) Evaluate $\displaystyle\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz \, dx \, dy$

$$\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz \, dx \, dy = \int_1^e \int_1^{\log y} \left[z \log z - z \right]_1^{e^x} dx \, dy$$

$$= \int_1^e \int_1^{\log y} \left[x e^x - e^x + 1 \right] dx \, dy$$

$$= \int_1^e \left[x e^x - e^x - e^x + x \right]_1^{\log y} dy$$

$$= \int_1^e \left[(x-2) e^x + x \right]_1^{\log y} dy$$

$$= \int_1^e \left(y \log y + \log y - 2y + e - 1 \right) dy$$

$$= \left[\left(\frac{y^2}{2} + y \right) \log y - \left(\frac{y^2}{4} + y \right) - y^2 + (e-1) y \right]_1^e$$

$$= \frac{e^2}{4} - 2e + \frac{13}{4}$$

$$= \frac{1}{4}\left(e^2 - 8e + 13 \right).$$

2) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$.

Let $\phi_1 = x^2+y^2+z^2-9=0$.

$\phi_2 = x^2+y^2-z-3=0$

$\Rightarrow \nabla\phi_1 = 2xi+2yj+2zk$

$\nabla\phi_2 = 2xi+2yj-k$

Let $\bar{n}_1 = \nabla\phi_1$ at $(2,-1,2) = 4i-2j+4k$

$\bar{n}_2 = \nabla\phi_2$ at $(2,-1,2) = 4i-2j-k$

$\bar{n}_1, \bar{n}_2$ are the normals of 2 surfaces at the point $(2,-1,2)$

θ be the angle b/w two surfaces.

$$\cos\theta = \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1||\bar{n}_2|}$$

$$= \frac{(4i-2j+4k)\cdot(4i-2j-k)}{\sqrt{16+4+16}\sqrt{16+4+1}}$$

$$= \frac{16+4-4}{6\sqrt{21}} = \frac{8}{3\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$$

3) Prove that $\text{grad}(\bar{a}\cdot\bar{b}) = (\bar{b}\cdot\nabla)\bar{a} + (\bar{a}\cdot\nabla)\bar{b} + \bar{b}\times \text{curl}\,\bar{a} + \bar{a}\times \text{curl}\,\bar{b}$

Consider

$$\bar{a}\times \text{curl}(\bar{b}) = \bar{a}\times(\nabla\times\bar{b})$$

$$= \bar{a}\times \sum i \times \frac{\partial \bar{b}}{\partial x}$$

$$= \sum \bar{a}\times\left(i\times\frac{\partial \bar{b}}{\partial x}\right)$$

$$= \sum\left\{\left(a\cdot\frac{\partial \bar{b}}{\partial x}\right)\bar{i} - (\bar{a}\cdot i)\frac{\partial \bar{b}}{\partial x}\right\}$$

$$= \sum \bar{i}\left(\bar{a}\cdot\frac{\partial \bar{b}}{\partial x}\right) - \left(\bar{a}\cdot\sum i\frac{\partial}{\partial x}\right)\bar{b}$$

$$\therefore \bar{a}\times \text{curl}\,\bar{b} = \sum i\left(\bar{a}\cdot\frac{\partial \bar{b}}{\partial x}\right) - (\bar{a}\cdot\nabla)\bar{b} \longrightarrow (1)$$

Similarly, $\bar{b}\times \text{curl}(\bar{a}) = \sum i\left(\bar{b}\cdot\frac{\partial \bar{a}}{\partial x}\right) - (\bar{b}\cdot\nabla)\bar{a} \longrightarrow (2)$

(1) + (2) $\Rightarrow$

$$\bar{a}\times\text{curl}\,\bar{b} + \bar{b}\times\text{curl}\,\bar{a} = \sum\bar{i}\left(\bar{a}\cdot\frac{\partial \bar{b}}{\partial x}\right) - (\bar{a}\cdot\nabla)\bar{b} + \sum i\left(\bar{b}\cdot\frac{\partial \bar{a}}{\partial x}\right) - (\bar{b}\cdot\nabla)\bar{a}$$

$$= \bar{a}\times\text{Curl}\,\bar{b} + \bar{b}\times\text{Curl}\,\bar{a} + (\bar{a}\cdot\nabla)\bar{b} + (\bar{b}\cdot\nabla)\bar{a}$$

$$= \sum i\left(\bar{a}\cdot\frac{\partial \bar{b}}{\partial x}\right) + \sum i\left(\bar{b}\cdot\frac{\partial \bar{a}}{\partial x}\right)$$

$$= \sum i\left(\bar{a}\cdot\frac{\partial \bar{b}}{\partial x} + \bar{b}\cdot\frac{\partial \bar{a}}{\partial x}\right)$$

$$= \sum i \frac{\partial}{\partial x}(\bar{a}\cdot\bar{b}).$$

$$= \nabla(\bar{a}\cdot\bar{b})$$

$$= \text{grad}(\bar{a}\cdot\bar{b}).$$

$$\therefore \text{grad}(\bar{a}\cdot\bar{b}) = (\bar{b}\cdot\nabla)\bar{a} + (\bar{a}\cdot\nabla)\bar{b} + \bar{b}\times \text{curl}\,\bar{a} + \bar{a}\times\text{curl}\,\bar{b}$$

4) Evaluate $\iint_S \bar{F}\cdot\bar{n}\,ds$ where $\bar{F} = 12x^2y\bar{i} - 3yzj + 2z\bar{k}$ and S is the portion of the plane $x+y+z=1$ included in the first Octant.

$$\bar{F} = 12x^2y\bar{i} - 3yz\bar{j} + 2z\bar{k}$$

W.K.T

$\nabla\phi$ is along the normal to surface $\phi(x,y,z)=0$

$\therefore$ Vector Perpendicular to $x+y+z=1$ is

$$\nabla(x+y+z) = \bar{i}+\bar{j}+\bar{k}$$

$$\text{Unit Normal } (\bar{n}) = \frac{\bar{i}+\bar{j}+\bar{k}}{\sqrt{1^2+1^2+1^2}}$$

$$= \frac{1}{\sqrt{3}}(\bar{i}+\bar{j}+\bar{k})$$

$$\bar{F}\cdot\bar{n} = \frac{1}{\sqrt{3}}\left[12x^2y - 3yz + 2z\right].$$

$$\left(\because x+y+z=1 \quad z = 1-x-y\right)$$

$$= \frac{1}{\sqrt{3}}\left[12x^2y - 3y(1-x-y) + 2(1-x-y)\right]$$

$$= \frac{1}{\sqrt{3}}\left[12x^2y + 3xy + 3y^2 - 2x - 5y + 2\right]$$

$$\therefore \iint_S \bar{F}\cdot\bar{n}\, ds = \iint_R \bar{F}\cdot\bar{n}\cdot\frac{dx\,dy}{|\bar{n}\cdot\bar{K}|} \quad (x\text{-}y \text{ plane}) \left(\bar{n}\cdot K = 1/\sqrt{3}\right)$$

$$= \iint_R (12x^2y + 3xy + 3y^2 - 2x - 5y + 2)\, dx\,dy$$

Since in XY plane $(z=0)$.

$$x+y+z=1$$

$$x+y+0=1$$

$$\boxed{\therefore y = 1-x}$$

Put $y=0$

$$\boxed{x=1}$$

Therefore y varies from $y=0$ to $y=1-x$

x varies from $x=0$ to $x=1$

$$\therefore \iint_S \bar{F}\cdot\bar{n}\, ds = \int_{x=0}^{1}\int_{y=0}^{1-x} (12x^2y + 3xy + 3y^2 - 2x - 5y + 2)\, dy\,dx$$

$$= \int_0^1 \left[12x^2\cdot\frac{y^2}{2} + 3x\frac{y^2}{2} + 3\frac{y^3}{3} - 2xy - 5\cdot\frac{y^2}{2} + 2y\right]_0^{1-x} dx$$

$$= \int_0^1 \left[\left(6x^2 + \frac{3}{2}x - \frac{5}{2}\right) y^2 + y^3 - 2xy + 2y \right]_0^{1-x} dx$$

$$= \int_0^1 \left[\left(6x^2 + \frac{3}{2}x - \frac{5}{2}\right)(1-x)^2 + (1-x)^3 - 2x(1-x) + 2(1-x) \right] dx$$

$$= \frac{1}{2}\int_0^1 (x^3 + 11x^2 - x - 8)\,dx = \frac{1}{2}\left[\frac{x^4}{4} + 11\frac{x^3}{3} - \frac{x^2}{2} - 8x \right]_0^1$$

$$= \frac{-55}{24}.$$

5) Verify Green's theorem in plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$ where C is the Region bounded by $y = \sqrt{x}$ and $y = x^2$

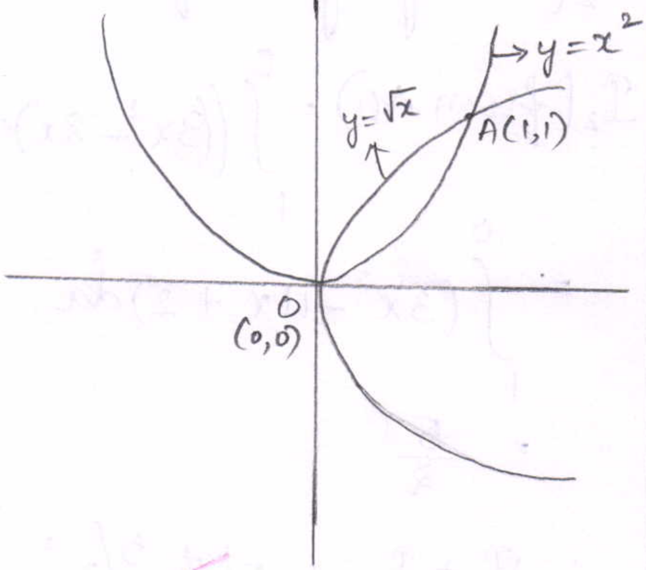
$M = 3x^2 - 8y^2$ and $N = 4y - 6xy$.

$\frac{\partial M}{\partial y} = -16y$ $\qquad$ $\frac{\partial N}{\partial x} = -6y$

By Green's theorem.

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy.$$

$$= \iint_R (16y - 6y)\,dx\,dy$$



$$= 10 \iint_R y\,dx\,dy.$$

$$= 10 \int_{x=0}^{1} \int_{y=x^2}^{\sqrt{x}} y\,dy\,dx$$

$$= 10 \int_{x=0}^{1} \left[\frac{y^2}{2}\right]_{x^2}^{\sqrt{x}} dx = 5\int_0^1 (x - x^4)\,dx$$

$$= 5\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = 5\left(\frac{1}{2} - \frac{1}{5}\right)$$

$$= \frac{3}{2}.$$

Verification :

$\int_{x=0}^{1} I_1$ (along $y=x^2$ from O to A). $y = x^2$, $dy = 2x\,dx$.

$$I_1 \text{ (from OA)} = \int_{x=0}^{1} (3x^2 - 8(x^2)^2)\,dx + (4x^2 - 6x(x^2))\,2x\,dx.$$

$$= \int_0^1 (3x^2 + 8x^3 - 20x^4)\,dx$$

$$= -1$$

I_2 (along $y^2 = x$ from A to O).

$$I_2(\text{from } AO) = \int_1^0 \left((3x^2 - 8x)dx + (4\sqrt{x} - 6x^{3/2})\frac{1}{2\sqrt{x}}dx\right)$$

$$= \int_1^0 (3x^2 - 11x + 2)dx$$

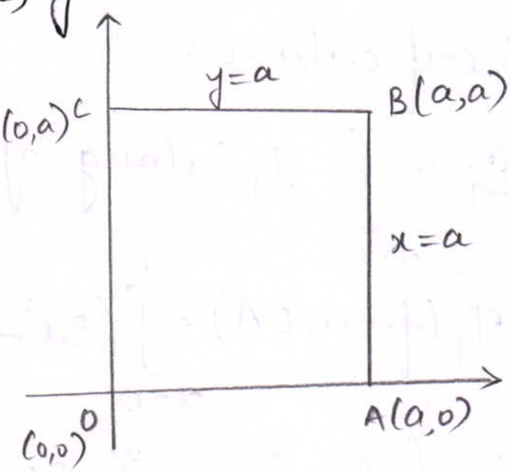
$$= \frac{5}{2}.$$

$\therefore I_1 + I_2 = -1 + 5/2 = 3/2$

$\therefore$ Verified Green's theorem

6) Verify stokes theorem for the function $F = x^2 i + xy j$ integrated round the square in the plane $z=0$ whose sides are along the lines $x=0, y=0, x=a, y=a$.

Given $\bar{F} = x^2 \hat{i} + xy \hat{j}$



By stokes theorem

$$\int_S (\nabla \times F) \cdot \bar{n}\, ds = \int_C \bar{F} \cdot dr$$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & 0 \end{vmatrix} = Ky.$$

$$L.H.S = \int_S (\nabla \times \bar{F}) \cdot \bar{n}\, ds$$

$$= \int_S y(\bar{n} \cdot K)\, ds = \int_S y\, dx\, dy$$

$$\therefore \int (\nabla \times \bar{F}) \cdot \bar{n}\, ds = \int_0^a \int_0^a y\, dy\, dx = \frac{a^3}{2}$$

$$R.H.S = \int_C F \cdot dr = \int_C (x^2 dx + xy\, dy)$$

But $\int \bar{F} \cdot dr = \int_{OA} \bar{F} \cdot dr + \int_{AB} \bar{F} \cdot dr + \int_{BC} \bar{F} \cdot dr + \int_{CO} \bar{F} \cdot dr$

i, Along OA $= y=0, z=0, dy=0, dz=0$

$$\int_{OA} \bar{F} \cdot dr = \int_0^a x^2 dx = \frac{a^3}{3}$$

ii) Along AB : $x=a, z=0, dx=0, dz=0$

$$\int_{AB} \bar{F} \cdot dr = \int_0^a ay\, dy = \frac{1}{2} a^3$$

iii) Along BC : $y=a, z=0, dy=0, dz=0$

$$\int_{BC} \bar{F} \cdot dr = \int_a^0 x^2 dx = \frac{-1}{3} a^3$$

iv) Along CO $= x=0, z=0, dx=0, dz=0$

$$\int_{CO} \bar{F} \cdot dr = \int_a^0 0\, dy = 0.$$

$$\therefore \int_C \bar{F}\cdot dr = \cancel{\frac{a^3}{3}} + \frac{a^3}{2} - \cancel{\frac{a^3}{3}} + 0$$

$$= \frac{a^3}{2}$$

Hence Verified

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TUTORIAL EVIDENCE

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

DEPARTMENT OF IT

TUTORIAL EVIDENCE

SUB: M2

YEAR/SEM: I/II **2016-17** **CLASS: IT**

TUTORIAL NUMBER	UNIT NUMBER	DATE	TOPIC	DESCRIPTION
1.	UNIT-I	7/1/17	Properties of laplace transform	Identifying and solving problems of laplace transforms.
2.		28/1	Application of L.T.	Applying laplace transform techniques to solve boundary and intial value problems
3	UNIT-II	14/2	Evaluation of integrals	• Solving integrals by using beta gamma functions
4.	UNIT-III	25/2	Conversion	Evaluating double integrals by transforming Cartesian to polar and vice versa
5.		25/3	Applications of double and triple integrals	• Finding area and volumes by using double and triple integrals
6.	UNIT-IV	7/4	Directional derivaties	• Evaluating the problems related to directional derivatives
7.	UNIT-V	22/4	Volume integrals	• Solving problems related to volume integrals

RESULT ANALYSIS TO IDENTIFY WEAK AND ADVANCED LEARNERS

I year IT

S.No	Roll No	Name of the student	Student performance upto previous	Student performance based on Internal	Over all Grade
1	16BD1A1201	A SANJANA	B+	A	A
2	16BD1A1202	ANSH KUMAR	C	B	B
3	16BD1A1203	B AMBICA	B+	A	A
4	16BD1A1204	BYREDDY MANASA REDDY	B	A	B+
5	16BD1A1205	BANDARU NIKAS	B+	A	A
6	16BD1A1206	BATHINI RAJ KUMAR	B	A	B+
7	16BD1A1207	BOKKA NISHIKANTH REDDY	C	A	B
8	16BD1A1208	BOLLAM LIBNAH PRATHEETH	B	A	B+
9	16BD1A1209	C TRILOK	F	B	C
10	16BD1A1210	BANDI SRI MEGHANA	C	B	B
11	16BD1A1211	DAKOJI MOUNIKA	C	A	B
12	16BD1A1212	DONKENA NAVEEN	B	A	B+
13	16BD1A1213	GANDIKOTA SHAFIL	B	A	B+
14	16BD1A1214	GARNEPUDI MAHESH CHANDRA TEJA	B+	B	B+
15	16BD1A1215	GUNDETI SRAVAN REDDY	C	A	B
16	16BD1A1216	HADIYA KOUSAR	B+	B	B+
17	16BD1A1217	GANTA HARSHIT REDDY	C	A	B
18	16BD1A1218	GAUTAMI JOGINAPALLY	F	A	B
19	16BD1A1219	JONNADA VIVEK	F	C	C
20	16BD1A1220	JAIDI SRUJAN REDDY	A	A	A
21	16BD1A1221	K DIXIT REDDY	F	B	C
22	16BD1A1222	K V S DATTATREYA RAMCHANDRA SU	F	A	C
23	16BD1A1223	KARISHMA SHARMA	A	A	A
24	16BD1A1224	KURRA SNEHA	B	A	B+
25	16BD1A1225	MAHAKALA MRUNAL	C	B	B
26	16BD1A1226	M NIKHIL REDDY	C	B	B
27	16BD1A1227	M PRERNA REDDY	C	B	B
28	16BD1A1228	MADHARI N NIKITHA	B	B	B
29	16BD1A1229	MADHU MITHA RAEPALA	B+	A	A
30	16BD1A1230	MOHD SHAFIQ AHMED	B	A	B+
31	16BD1A1231	NAMPALLY DIVYAANI KAUSHAL	B+	A	A
32	16BD1A1232	NARRA SRI KAVYA REDDY	A	A	A
33	16BD1A1233	NAYANI VENKATESH REDDY	F	B	C
34	16BD1A1234	NIDHI BANG	A	A	A
35	16BD1A1235	NIRMALA AKSHAY	C	A	B
36	16BD1A1236	POLAMANI VAMSHI KRISHNA	F	B	C
37	16BD1A1237	PABBOJU VAISHNAVI	A	A	A
38	16BD1A1238	PIYUSH HARSHE	A	A	A
39	16BD1A1239	PURVA SHAH	A	A	A
40	16BD1A1240	RACHAKONDA PREETHI	A	A	A
41	16BD1A1241	REDDIPALLI SRAVYA	B	B	B

42	16BD1A1242	RIDDHI MEDAKKAR	B	B	B
43	16BD1A1243	POLAMAINA RUTHVIK YADAV	F	C	C
44	16BD1A1244	S SHIVA RANGA NAYAK	C	C	C
45	16BD1A1245	SANGEPU NIHAAL	B+	B	B+
46	16BD1A1246	SIDDI CHANDU	A	A	A
47	16BD1A1247	SURUGU DILEEP KUMAR	C	B	B
48	16BD1A1248	T CHANDRA LOHIT REDDY	B+	B	B+
49	16BD1A1249	THAKUR SAI SHYAM SINGH	B	A	B+
50	16BD1A1250	THARIGOPULA NIKHIL PRASHANTH	B+	A	A
51	16BD1A1251	THOOM SRI HARSHA	B	A	B+
52	16BD1A1252	VARALA SANDEEP REDDY	B+	A	A
53	16BD1A1253	VARIKOTI SAKETH CHAKRAVARTHY	F	A	C
54	16BD1A1254	Y V MANOJ DUTT	C	A	B
55	16BD1A1255	YANDAPALLY SRI DIVYA	B	A	B+
56	16BD1A1256	YENUGULA KANISHK	B+	A	A
57	16BD1A1257	M PREETHAM KUMAR	C	B	B
58	16BD1A1258	RAMAGIRI SINDHUJA	C	B	B
59	16BD1A1259	SARA SRIKANTH	F	C	C
60	16BD1A1260	YEDLA SAI PRANITHA	B+	**B**	A

COURSE ASSESSMENT

KESHAVA MEMORIAL INSTITUTE OF TECHNOLOGY

Department of Information Technology

Course Outcome Attainment

Name of the faculty : Dr.T.V.A.P.SASTRY — Academic Year 2016-17

Branch & Section: IT — Exam: I Internal

Subject: **Mathematics-2** — Year: I — Semester: II

S.No	HT No.	Question No.												Obj1	A1
		1A	1B		2A	2B		3A	3B		4A	4B			
Max. Marks ==>		**2**	**3**		**2**	**3**		**2**	**3**		**2**	**3**		**10**	**5**
1	16BD1A01201	2	2		1.5	1		2	0		2	3		6	5
2	16BD1A01202	3	1								0			4	5
3	16BD1A01203	2	3		2						2	3		9	5
4	16BD1A01204	2	3		2	3								4	5
5	16BD1A01205	2	3					2	2					8	5
6	16BD1A01206	2	3		2	3								8	5
7	16BD1A01207	2	3								2	3		6	5
8	16BD1A01208	2	3								2	3		5	5
9	16BD1A01209	2	2		0						2	2		6	5
10	16BD1A01210	0	2		0	0		2	1		2	2		4	5
11	16BD1A01211							2	2		2	3		5	5
12	16BD1A01212	2	3					2			2	3		7	5
13	16BD1A01213	2	3		2	1					2	3		7	5
14	16BD1A01214	2	3		1	0					0	0		5	5
15	16BD1A01215	2	3		2	1								5	5
16	16BD1A01216	1	2		0	2					1	2		4	5
17	16BD1A01217	2	3					2			1	0		6	5
18	16BD1A01218	2	3					1				3		6	5
19	16BD1A01219							1.5				2,5		5	5
20	16BD1A01220	2	3		1.5	2		2	3		2	3		9	5
21	16BD1A01221	2	3		1									8	5
22	16BD1A01222	2	2								2			9	5
23	16BD1A01223	2	3		2	2		2			2	3		9	5
24	16BD1A01224	2	1		2	2								7	5
25	16BD1A01225	0	0		1	1					2			6	5
26	16BD1A01226	2	3					1						4	5
27	16BD1A01227	2	0		0			0	0					4	5
28	16BD1A01228	0	0		0			3			1			4	5
29	16BD1A01229	2	0.5		2	3					2	2		6	5
30	16BD1A01230	2	3								2	3		5	5
31	16BD1A01231	2	3					2	3		2	3		9	5
32	16BD1A01232	2	3		1	2		2	3		2	3		7	5
33	16BD1A01233	2	3			3			2					3	5
34	16BD1A01234	2	3		0						2	3		6	5
35	16BD1A01235	2	3								2	3		6	5
36	16BD1A01236	2	1		1									7	5
37	16BD1A01237	2	3		2	2		2	3					8	5
38	16BD1A01238	2	3					2	3		2			7	5

39	16BD1A01239	2	3		2						2	2		7	5
40	16BD1A01240	1.5	2.5		2	3		2	3		2	2		9	5
41	16BD1A01241	2			1	3		0						4	5
42	16BD1A01242	2			0						0	1		2	5
43	16BD1A01243	0												2	5
44	16BD1A01244				0	0			2					2	5
45	16BD1A01245	2	3								2	3		2	5
46	16BD1A01246	2	2					2	3		2	3		6	5
47	16BD1A01247	2	2					1						6	5
48	16BD1A01248	2	3					2	2					6	5
49	16BD1A01249	2	3					3	1					4	5
50	16BD1A01250	2	3								2	3		7	5
51	16BD1A01251	2	3								2	3		5	5
52	16BD1A01252	2	3								2	3		8	5
53	16BD1A01253	2	3								2	3		3	5
54	16BD1A01254	2	3		2	3								3	5
55	16BD1A01255	2	3		2	3		3						3	5
56	16BD1A01256	1	2		1	3								6	5
57	16BD1A01257	2	1.5		1.5	2								5	5
58	16BD1A01258	0	0		2	0.5		3						4	5
59	16BD1A01259	0						0	0					3	5
60	16BD1A01260	2	3		2	3		0						4	5
	SUM	100.5	128.5	0	41.5	48.5	0	46.5	33	0	59	76	0	335	300
	COUNT	57	53	0	33	25	0	27	17	0	34	30	0	60	60
	AVERAGE	1.763	2.425		1.26	1.94		1.72	1.94		1.74	2.53		5.5833	5

CO Mapping with Exam Questions:

CO - 1	**Y**	**Y**		Y	**Y**								Y	Y
CO - 2							**Y**	**Y**		**Y**	**Y**		Y	Y
CO - 3														
CO - 4														
CO - 5														
Students Scored >Target %	50	45	60	25	17	60	23	12	60	31	28	60	40	60
% Students Scored >Target %	88%	85%		76%	68%		85%	71%		91%	93%		67%	100%

CO Attainment based on Exam Questions:

CO - 1	88%	85%		76%	68%								67%	100%
CO - 2							85%	71%		91%	93%		67%	100%
CO - 3														
CO - 4														
CO - 5														

CO	Subj	obj	Asgn	Overall	Level
CO-1	79%	67%	100%	82%	3
CO-2	85%	67%	100%	84%	3
CO-3					
CO-4					
CO-5					

Attainment Leve	
1	<40%
2	40-60%
3	>60%

Overall Course Attainment = 3

Keshav Memorial Institute of Technology

Department of Information Technology

Course Outcome Attainment

Name of the faculty : **Dr. T.V.A.P.SASTRY** Academic Year : **2016-17**

Branch & Section : **IT** Year : **1** Semester : **1**

Subject : **M 2** Exam : **II Internal**

S.No	HT No.	Question No.												Obj1	A1
		1A	1B	1C	2A	2B	2C	3A	3B	3C	4A	4B	4C		
Marks ==>		1.0	4.0		2.5	2.5		2.0	3.0		5.0				
1	16BD1A1201							2.0	3.0		5.0				
2	16BD1A1202					2.0		2.0							
3	16BD1A1203					2.0		2.0	3.0						
4	16BD1A1204					2.5		1.0	1.0						
5	16BD1A1205	1.0	0.0		1.0	2.5		2.0	3.0						
6	16BD1A1206	1.0						2.0	3.0		1.0				
7	16BD1A1207					0.0					1.0				
8	16BD1A1208					1.5		2.0	2.5						
9	16BD1A1209		0.0			2.0		2.0	1.0						
10	16BD1A1210					1.0		1.5			5.0				
11	16BD1A1211		0.0			2.5		1.0	1.5						
12	16BD1A1212					2.5		2.0	2.5						
13	16BD1A1213	1.0	0.0			2.5		2.0	3.0		0.0				
14	16BD1A1214							1.5	1.5		2.0				
15	16BD1A1215	0.5				2.5		2.0	2.0						
16	16BD1A1216					2.0		2.0							
17	16BD1A1217	1.0	0.0			2.5		2.0							
18	16BD1A1218	1.0	0.0			2.5		2.0							
19	16BD1A1219	1.0				1.0									
20	16BD1A1220				0.0	2.5		2.0	2.0		5.0				
21	16BD1A1221							2.0	2.0						
22	16BD1A1222							2.0			1.0				
23	16BD1A1223					1.0		2.0	3.0						
24	16BD1A1224	1.0	0.0					2.0	1.0		2.0				
25	16BD1A1225	0.5						1.0			0.5				
26	16BD1A1226					2.0		2.0	1.0						
27	16BD1A1227					2.5		2.0	2.5		0.5				
28	16BD1A1228					2.5		2.0							
29	16BD1A1229					2.5		2.0	1.5						
30	16BD1A1230					2.5		2.0	3.0						
31	16BD1A1231	1.0	0.0					2.0	3.0		5.0				
32	16BD1A1232	1.0	0.0		2.5	2.5		2.0	1.5						
33	16BD1A1233	0.0						0.0							
34	16BD1A1234	1.0	4.0					2.0	3.0						
35	16BD1A1235	1.0	4.0					2.0	3.0						
36	16BD1A1236	1.0	4.0			2.5									
37	16BD1A1237	0.5	1.5			2.5		2.0							
38	16BD1A1238							2.0	2.0		5.0				
39	16BD1A1239							2.0	2.5		5.0				
40	16BD1A1240	1.0	0.0			2.0		2.0	3.0						
41	16BD1A1241				2.0	2.0					5.0				
42	16BD1A1242	1.0	3.0					2.0			2.0				
43	16BD1A1243	1.0	1.0					2.0	0.0						
44	16BD1A1244					2.0		2.0	3.0						
45	16BD1A1245		4.0			2.5									

46	16BD1A1246	1.0	1.0								5.0				
47	16BD1A1247	1.0						2.0	3.0						
48	16BD1A1248	1.0	1.0								1.0				
49	16BD1A1249	1.0						0.0							
50	16BD1A1250							2.0	3.0		5.0				
51	16BD1A1251							2.0	0.0		1.0				
52	16BD1A1252					2.5		2.0	3.0						
53	16BD1A1253					2.5		2.0	3.0						
54	16BD1A1254	1.0	2.0			2.5									
55	16BD1A1255	1.0	4.0		2.0	2.5									
56	16BD1A1256					1.5		2.0	2.0		1.0				
57	16BD1A1257					2.5		2.0	3.0						
58	16BD1A1258	1.0						1.0	3.0						
59	16BD1A1259							1.0							
60	16BD1A1260					1.5		2.0	3.0		5.0				
61															
62															
63															
64															
65															
66															
67															
68															
	SUM	23.5	29.5	0	7.5	78.5	0	92	87	0	63	0	0	0	0
	COUNT	26	21	0	5	37	0	51	38	0	22	0	0	0	0
	AVERAGE	0.9	1.4		1.5	2.1		1.8	2.3		2.9				

CO Mapping with Exam Questions:

CO - 1														
CO - 2														
CO - 3	**Y**												**Y**	**Y**
CO - 4				**Y**			**Y**	**Y**					**Y**	**Y**
CO - 5										**Y**			**Y**	**Y**

Students Scored >Target %	25	7	0	3	33	0	49	32	0	10	0	0	68	68
% Students Scored >Target %	96%	33%		60%	89%		96%	84%		45%				

CO Attainment based on Exam Questions:

CO - 1														
CO - 2														
CO - 3	96%													
CO - 4				60%			96%	84%						
CO - 5										45%				

CO	Subj	obj	Asgn	Overall	Level
CO-1					
CO-2					
CO-3	96%			96%	3
CO-4	80%			80%	3
CO-5	45%			45%	2

Attainment Level	
1	<40%
2	40-60%
3	>60%

Overall Course Attainment = 2.6667

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY
(Approved by AICTE & Govt of T.S and Affiliated to JNTUH)
3-5-1026, Narayanaguda, Hyderabad-29. Ph: 040-23261407

Department Of Information Technology
I B.Tech II Sem Regular Examination 2016-17

SUBJECT	FACULTY NAME	STUDENTS APPEARED	STUDENTS PASSED	STUDENTS FAILED	PASS %
EG	MR. PRAVEEN KUMAR PATIL	60	52	8	86.66
CP in C	DR. RAMAKANTH MAHANTHY	60	46	14	76.66
EP-2	K. VENKATESWARA RAO	60	39	21	65
M-2	DR.T.V.A.P.SASTRY	60	53	7	88.33
M-3	MD. YOUNUS	60	57	3	95
EP LAB	K.V. RAO & MR. O. RAGHAVENDRA PRASAD	60	60	0	100
EC LAB	MS. HEMANGI JOSHI & MR. GK.SRINIVAS	60	60	0	100
CP LAB	DR. RAMAKANTH MAHANTHY/ MS.SAMATHA/ MS.SARIKA	60	60	0	100

OVERALL REPORT

	NO.OF STUDENTS
Distinction	0
1 st class	6

CLASS TOPPERS

S.NO	REG.NO	STUDENT NAME	%
1.	16BD1A1246	SIDDI CHANDU	69
2.	16BD1A1240	RACHAKONDA PREETHI	68
3.	16BD1A1205	BANDARU NIKAS	66
4.	16BD1A1231	N DIVYAANI KAUSHAL	66

Committee In Charge **HOD** **Principal**

Keshav Memorial Institute of Technology

Department of Information Technology

Course Outcome Attainment

Name of the faculty : Dr. T.V.A.P.SASTRY
Academic Year : 2016-17

Branch & Section : IT
Exam : Overall

Subject : M 2
Year: 1
Semester : 1

Course Outcomes	1st Internal Exam	2nd Internal Exam	University Exam
CO1	3		2
CO2	3		2
CO3		3	2
CO4		3	2
CO5		2	2

Attainment level of Course Outcomes

	Course Outcomes	Attainment Level
CO1	Use Laplace transform techniques for solving Diferential Equations(PO1), (PO2)	2
CO2	Evaluate Integrals using Beta and Gamma functions (PO1), (PO3)	2
CO3	Evaluate Multiple Integrals and can apply these concepts to find areas, volumes,moment of inertia etc of regions on a plane or in space(PO1), (PO2)	2
CO4	Distinguish between the line and surface and volume integrals and convert them from one to another(PO1), (PO3)	2
CO5	Transform Surface Integrals to volume Integrals (PO1), (PO2)	2

Average : 2.2

Overall course attainment level **2**

Faculty Signature

Keshav Memorial Institute of Technology

Department of Computer Science Engineering

Programme Outcome Attainment

Name of the faculty : **Dr. T.V.A.P.SASTRY** Academic Year : **2016-17**

Branch & Section : **IT** Year : 1 Sem : **1**

Subject : **M 2**

CO	Mid-1	Mid-2	AVG	Univ	DIRECT	INDIRECT	OVERALL
CO1	3		3	2	2.25		1.8
CO2	3		3	2	2.25		1.8
CO3		3	3	2	2.25		1.8
CO4		3	3	2	2.25		1.8
CO5		2	2	2	2		1.6
CO6	0	0	0	0	0		0
ATTAINMENT	3	2.66667	2.8	2	2.20833		

CO-PO mapping

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1	H	M												
CO2	H		M											
CO3	H	M												
CO4	H		M											
CO5	H	M												

KESHAV MEMORIAL INSTITUTE OF TECHNOLOGY

Department of Information Technology

Program Specific Outcome Attainment

Name of Faculty: Dr. T.V.A.P.SASTRY Academic Year: 2016-17

Branch & Section: IT Year : 1

Subject: M 2 Sem : 1

Course outcome attainment

CO	IstMid	IIndMid	Univ	DIRECT	INDIRECT	OVERALL
CO1	3		2	2.25		1.8
CO2	3		2	2.25		1.8
CO3		3	2	2.25		1.8
CO4		3	2	2.25		1.8
CO5		2	2	2		1.6
average	3	2.6666667	2	2.2		1.76

CO-PSO mapping

	PSO1	PSO2
CO1	0	0
CO2	0	0
CO3	0	0
CO4	0	0
CO5	0	0
average	0	0
attainment		

Faculty

Keshav Memorial Institute of Technology

Narayanaguda. Hyderabad 500027

Analysis of Weak and Advanced Learners

Name of the faculty : **Dr. T.V.A.P.SASTRY** Academic Year : **2016-17**

Branch & Section : **IT** Exam : Overall

Subject : **M 2** Year : **1** Semester **1**

S. No	Roll No	Name of the student	Student performance Based on Internal Exam marks			
			Up to previous semester	MID1 Marks	MID2 Marks	Overall Grade
1	16BD1A1201	A SANJANA	B+	C	C	C
2	16BD1A1202	ANSH KUMAR	C	B	C	C
3	16BD1A1203	B AMBICA	A	A	C	B
4	16BD1A1204	BYREDDY MANASA REDDY	C	B	C	C
5	16BD1A1205	BANDARU NIKAS	B+	B	C	B
6	16BD1A1206	BATHINI RAJ KUMAR	C	A	C	C
7	16BD1A1207	BOKKA NISHIKANTH REDDY	C	B	C	C
8	16BD1A1208	BOLLAM LIBNAH PRATHEETH	C	A	C	C
9	16BD1A1209	C TRILOK	B	A	C	B
10	16BD1A1210	BANDI SRI MEGHANA	C	C	C	C
11	16BD1A1211	DAKOJI MOUNIKA	C	C	C	C
12	16BD1A1212	DONKENA NAVEEN	C	C	C	C
13	16BD1A1213	GANDIKOTA SHAFIL	B+	C	C	C
14	16BD1A1214	GARNEPUDI MAHESH CHANDRA TEJA	B	B	C	B
15	16BD1A1215	GUNDETI SRAVAN REDDY	C	B	C	C
16	16BD1A1216	HADIYA KOUSAR	B	B	C	B
17	16BD1A1217	GANTA HARSHIT REDDY	C	B	C	C
18	16BD1A1218	GAUTAMI JOGINAPALLY	F	A	C	C
19	16BD1A1219	JONNADA VIVEK	F	C	C	C
20	16BD1A1220	JAIDI SRUJAN REDDY	A	B	C	B
21	16BD1A1221	K DIXIT REDDY	F	B	C	C
22	16BD1A1222	K V S DATTATREYA RAMCHANDRA SUBBA RA	F	C	C	C
23	16BD1A1223	KARISHMA SHARMA	B+	B	C	C
24	16BD1A1224	KURRA SNEHA	C	B	C	C
25	16BD1A1225	MAHAKALA MRUNAL	F	B	C	C
26	16BD1A1226	M NIKHIL REDDY	C	A	C	C
27	16BD1A1227	M PRERNA REDDY	F	B	C	C
28	16BD1A1228	MADHARI N NIKITHA	C	B	C	C
29	16BD1A1229	MADHU MITHA RAEPALA	B+	C	C	C
30	16BD1A1230	MOHD SHAFIQ AHMED	C	C	C	C
31	16BD1A1231	NAMPALLY DIVYAANI KAUSHAL	B+	A	C	B
32	16BD1A1232	NARRA SRI KAVYA REDDY	A	A	C	A
33	16BD1A1233	NAYANI VENKATESH REDDY	F	B	C	C
34	16BD1A1234	NIDHI BANG	A+	B	C	B

S. No	Roll No	Name of the student	Student performance Based on Internal Exam marks			
			Up to previous semester	MID1 Marks	MID2 Marks	Overall Grade
35	16BD1A1235	NIRMALA AKSHAY	B	C	C	C
36	16BD1A1236	POLAMANI VAMSHI KRISHNA	C	B	C	C
37	16BD1A1237	PABBOJU VAISHNAVI	A	A	C	B
38	16BD1A1238	PIYUSH HARSHE	A	B	C	B
39	16BD1A1239	PURVA SHAH	B+	A	C	B
40	16BD1A1240	RACHAKONDA PREETHI	A+	A	C	B
41	16BD1A1241	REDDIPALLI SRAVYA	F	C	C	C
42	16BD1A1242	RIDDHI MEDAKKAR	B	B	C	B
43	16BD1A1243	POLAMAINA RUTHIVIK YADAV	C	B	C	C
44	16BD1A1244	S SHIVA RANGA NAYAK	F	C	C	C
45	16BD1A1245	SANGEPU NIHAAL	B	B	C	B
46	16BD1A1246	SIDDI CHANDU	A+	A	C	B
47	16BD1A1247	SURUGU DILEEP KUMAR	F	C	C	C
48	16BD1A1248	T CHANDRA LOHIT REDDY	A+	A	C	C
49	16BD1A1249	THAKUR SAI SHYAM SINGH	C	C	C	C
50	16BD1A1250	THARIGOPULA NIKHIL PRASHANTH	B+	A	C	B
51	16BD1A1251	THOOM SRI HARSHA	C	C	C	C
52	16BD1A1252	VARALA SANDEEP REDDY	B	C	C	C
53	16BD1A1253	VARIKOTI SAKETH CHAKRAVARTHY	C	C	C	C
54	16BD1A1254	Y V MANOJ DUTT	B	A	C	B
55	16BD1A1255	YANDAPALLY SRI DIVYA	B	A	C	B
56	16BD1A1256	YENUGULA KANISHK	B	B	C	B
57	16BD1A1257	M PREETHAM KUMAR	C	B	C	C
58	16BD1A1258	RAMAGIRI SINDHUJA	C	A	C	C
59	16BD1A1259	SARA SRIKANTH	F	A	C	C
60	16BD1A1260	YEDLA SAI PRANITHA	C	B	C	C
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ATTENDANCE REGISTER